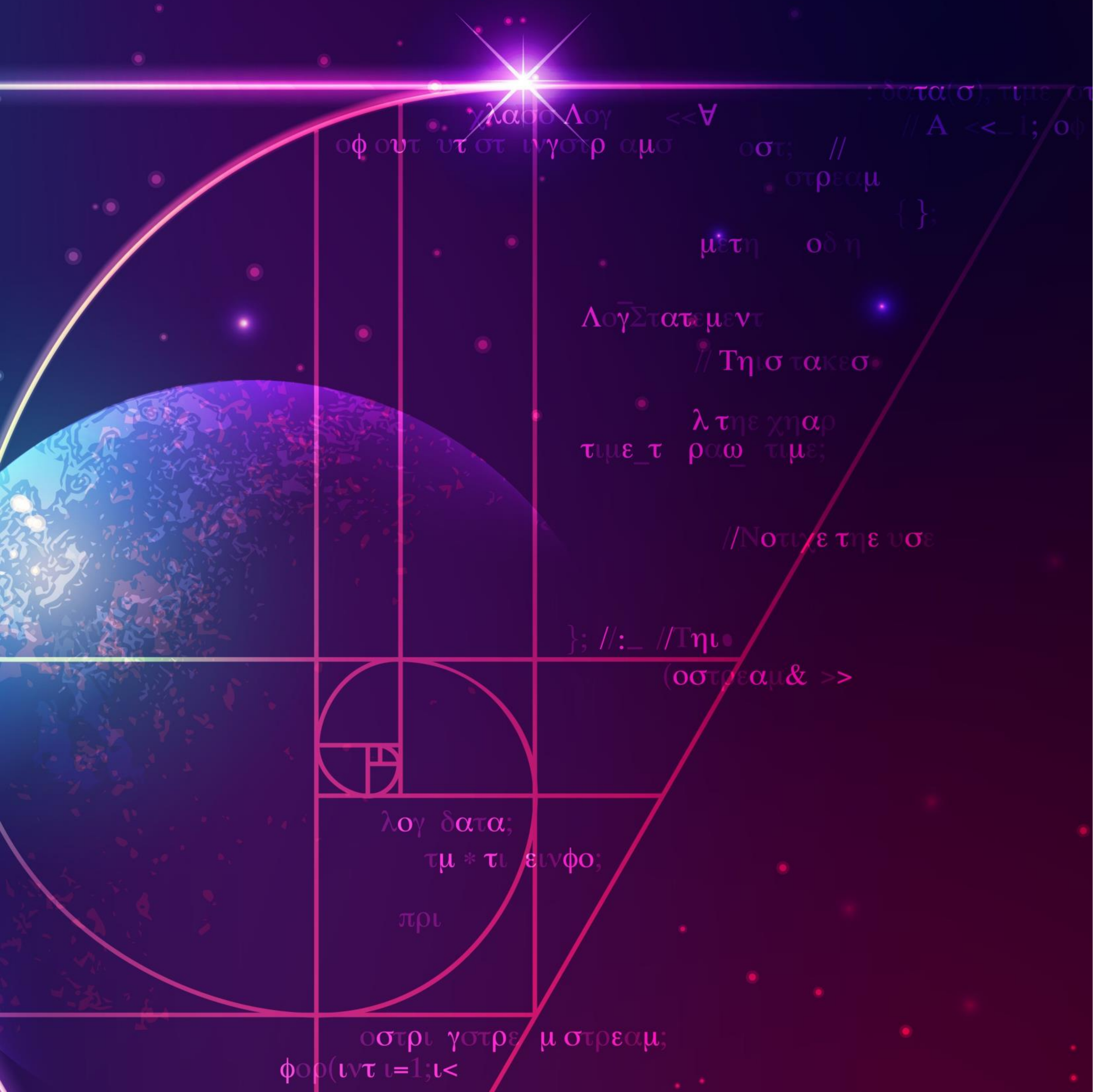
CHAPTER (4)

LINES



The line is one-dimensional figure, which has length but no width. In geometry, there are different types of lines such as horizontal and vertical, parallel and perpendicular. The change in y-value divided by the change if x-value in the coordinate grid is called the ***slope***. The slope of a line is a measure of the incline of the line. The slope is a number which indicates both direction and steepness of the line.

The slope of the line that passes through the points $(x_1, y_1)(x_2, y_2)$ can be found be the rule $\frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$

Example

Find the slope of the line that passes through the points $(3,4)$ and $(-1,5)$

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5-4}{-1-3}$$
$$= \frac{1}{-4}$$
$$= -\frac{1}{4}$$

4.1 Find the slope of the line that passes through the points $(2,3),(-4,-2)$

A $\frac{6}{5}$ **B** $-\frac{6}{5}$

C $\frac{5}{6}$ **D** $-\frac{5}{6}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2-3}{-4-2}$$
$$= \frac{-5}{-6}$$
$$= \frac{5}{6}$$

➢ B

4.2 Find k such that the slope of line which passes through the points $(-k,-7)$ and $(1,9)$ equals 4:

A 3 **B** – 3

C 5 **D** – 5

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$4 = \frac{9-(-7)}{1-(-k)}$$
$$4 = \frac{9+7}{1+k}$$
$$4(1+k) = 16$$
$$1+k = 4$$
$$k = 3$$

➢ A

Forms of Equation of a line

Point slope form	$y - y_1 = m(x - x_1)$
Slope Intercept form	$y = mx + b$

- The vertical line has an undefined slope
- The slope of a horizontal line is zero
- Two parallel lines have the same slope
- The product of the slope of two perpendicular lines is

$$(-1) \qquad m_1.m_2 = -1 \to m_1 = \frac{-1}{m_2}$$

4.3 Find the slope of the line perpendicular to $y = -2x + 7$

A $\frac{1}{2}$ **B** $-\frac{1}{2}$

C -2 **D** 2

The slope of the given line is -2

$$m_1.m_2 = -1$$
$$m_2 = \frac{-1}{m_1}$$
$$m_2 = \frac{-1}{-2}$$
$$= \frac{1}{2}$$

Or simply, the slope of a perpendicular line to a given line is negative reciprocal

The slope of the given line is

$$\frac{-2}{1} \to -\left(\frac{1}{-2}\right) = \frac{1}{2}$$

➢ A

4.4 Describe the two lines $l_1 : y = -2x + 4$
$l_2 : 2y = 16x + 8$

A Two parallel lines

B Both lines have the same y- intercept

C Two perpendicular lines

D Both lines have the same x- intercept

Write the equations in the form of $y = mx + b$

$l_1 = y = -2x + 4$

$m_1 = -2$

$y\text{-intercept} = 4$

$l_2 = \frac{2y}{2} = \frac{16x}{2} + \frac{8}{2}$

$y = 4x + 4$

$m_2 = 4$

$y\text{-intercept} = 4$ ➢ **B**

4.5 Find the equation of the line if its slope is 5 and the y - intercept is – 4

A $y = 5x + 4$ **B** $y = -4x + 5$

C $y = -5x - 4$ **D** $y = 5x - 4$

$y = mx + b$

m is the slope = 5

b is the y-intercept = (– 4)

$y = 5x + (-4)$

$= 5x - 4$ ➢ **D**

4.6 Find the equation of the line that passes through the point $(8, 0)$ and its slope is 4

A $y = 2x + 8$ **B** $y = 2x - 8$

C $y = 4x - 32$ **D** $y = 4x + 32$

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x - 8)$

$y = 4x - 32$ ➢ **C**

4.7 Find the equation of the line passes through the points $(-3, 4), (0, 2)$

A $\frac{2}{3}x + 2$ **B** $\frac{2}{3}x - 2$

C $-\frac{2}{3}x + 2$ **D** $-\frac{2}{3}x - 2$

Step 1:
Find the slope

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4 - 2}{-3 - 0}$

$= -\frac{2}{3}$

Step 2:
Choose one point → $(0, 2)$

$y - y_1 = m(x - x_2)$

$y - 2 = -\frac{2}{3}(x - 0)$

$y - 2 = -\frac{2}{3}(x)$

$y = -\frac{2x}{3} + 2$

Special Case:
Since one of the points has x-coordinate equals zero then the y-coordinate will be b the y-intercept

$y = mx + b$

$y = -\frac{2x}{3} + 2$

➢ **C**

4.8 Find the equation of line l perpendicular to the line $y = -\frac{1}{5}x + 4$ at the point $(0, 4)$

A $5x + 4$ **B** $5x - 4$

C $-5x + 4$ **D** $-5x - 4$

The slope of the line $y = -\frac{1}{5}x + 4$ is $-\frac{1}{5}$

Therefore, the negative reciprocal is 5 is the slope of the line l

At the point $(0, 4)$ means the y-intercept is 4 and by using $y = mx + b$ the equation is $y = 5x + 4$ ➢ **A**

4.9 Find the equation of the line l_1 parallel to the line l_2 passes through $(3,4)$ and $(2,0)$

A $y=4x+7$ **B** $y=\frac{1}{4}x+7$

C $y=-4x+7$ **D** $y=-\frac{1}{4}x+7$

Step 1:
Find the slope of line l_2

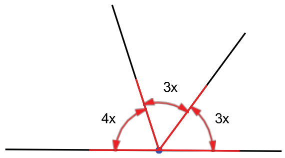
$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{4-0}{3-2}$$
$$=4$$

Step 2:
Since the two lines are parallel; $l_1 \parallel l_2$
then $m_1=m_2$

and the y-intercept could be any other point

A

4.10 Find x in the figure



A 20 **B** 18
C 1.8 **D** 30

Since the three angles are adjacent on a line and sharing the same vertex then they are supplementry

$$4x+3x+3x=180$$
$$10x=180$$
$$x=18$$

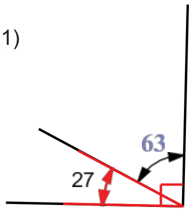
B

Parallel Lines and Angles

Recall that:

(1) Complementary angles are two angles with a sum of 90^o
(2) Supplementary angles are two angles with a sum of 180^o
(3) Vertical angles are two angles opposite to each other, they are always equal
(4) Angles around a point add up to 360^o

(1)



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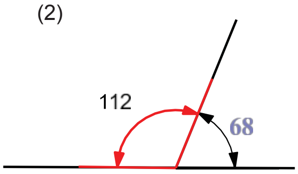

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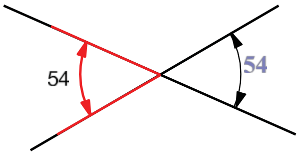

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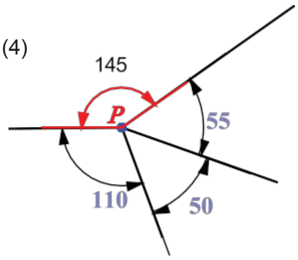
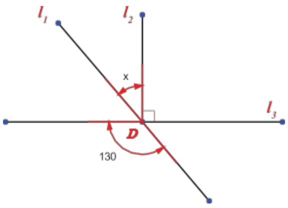


Example
If $\angle 1$ and $\angle 2$ are two complementary angles, and $m\angle 1=35$ then find $m\angle 2$

Solution:
Since $\angle 1$ and $\angle 2$ are complementary then

$$m\angle_1+m\angle_2=90$$
$$35+m\angle_2=90$$
$$m\angle_2=55$$

4.11 Find x in the figure



A 40 **B** 50
C 30 **D** 45

Focus on the two vertical angles formed by l_1 and l_3

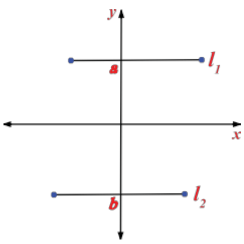
$$x+90=130$$
$$x=40$$

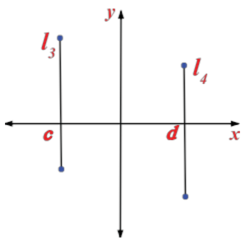
A

Distance Between Two Parallel Lines

If $l_1: y=a$ and $l_2: y=b$; and $l_1 \parallel l_2$ then the distance from l_1 to l_2 is $|a-b|$

If $l_3: x=l_4: x=d$, and $l_3 \parallel l_4$ then the distance from l_1 to l_2 is $|c-d|$





Example
Find the distance between the lines $y=-4$ and $y=-7$

$$d=|a-b|$$
$$=|-7-(-4)|$$
$$=|-3|=3$$

Example

Find the distance between the lines $x=0$ and $y=-7$

$$d=|0-7|$$
$$=|-7|=7$$

Note that because the distance cannot be negative it doesn't matter which one is the minuend and which one is the subtrahend in the absolute expression

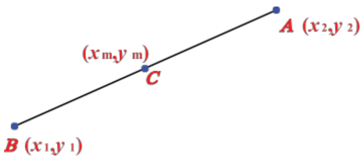
such that $|0-7|=|7-0|$

Midpoint

- For the two points $p_1(x_1,y_1)$ and $p_2(x_2,y_2)$ the coordinates of their midpoint

$$p_m(x_m,y_m)=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$$

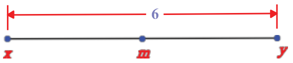
- If the point C is a midpoint of $\overline{AB}$ then $\overline{AC}\cong\overline{BC}$



Example

If m is midpoint of $\overline{xy}$ and $xy=6$ then find xm

Since m is midpoint of $\overline{xy}$



$$\overline{xm}\cong\overline{my}$$
$$xm=my$$
$$xy=xm+my$$
$$=xm+xm$$
$$6=2xm$$
$$3=xm$$

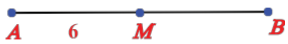
Example

Find the midpoint between $A(4,8)$ and $B(2,12)$

Solution:

$$midpoint=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$$
$$=(\frac{4+2}{2},\frac{8+12}{2})$$
$$=(3,10)$$

4.12 If M is the midpoint of $\overline{AB}$ and $AM=6$ then find AB



A 12   B 3

C 2   D 24

Since M is the midpoint of AB

l_1 l_3

$$\overline{AM}\cong\overline{MB}$$
$$AM=MB=6$$
$$AB=AM+MB$$
$$AB=6+6=12$$

4.13 If the end point coordinates of the line are $(-1,-2)$ and the coordinates of the midpoint are $(3,1)$, then what is the coordinates of the other endpoint?

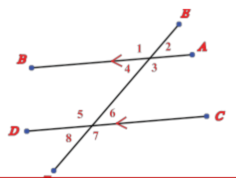
A $(7,4)$ B $(1,-\frac{1}{2})$

C $(-2,\frac{3}{2})$ D $(4,1)$

$$\text{midpoint } m=(xm,ym)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$p_1(x_1,y_1)\to(-1,-2)$$

$$x_m=\frac{x_1+x_2}{2} \qquad y_m=\frac{y_1+y_2}{2}$$
$$3=\frac{-1+x_2}{2} \qquad 1=\frac{-2+y_2}{2}$$
$$6=-1+x_2 \qquad 2=-2+y_2$$
$$7=x_2 \qquad 4=y_2$$
$$\to p_2(7,4)$$ A

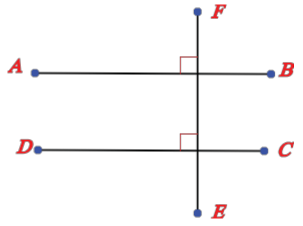
Parallel Lines and Angles



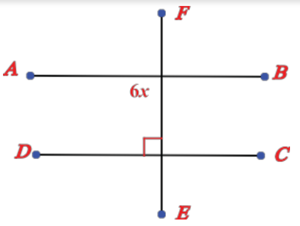
Alternate angles	Corresponding angles	Same Side
Interior		Interior
$\angle_3=\angle_5$	$\angle_1=\angle_5$	$\angle_4+\angle_5=180$
$\angle_4=\angle_6$	$\angle_2=\angle_6$	$\angle_3+\angle_6=180$
Exterior	$\angle_8=\angle_4$	Exterior
$\angle_1=\angle_7$	$\angle_7=\angle_3$	$\angle_1+\angle_8=180$
$\angle_2=\angle_8$		$\angle_2+\angle_7=180$

Perpendicular Transversal

If a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other



4.14 Find x



A 90 **B** 30

C 15 **D** 20

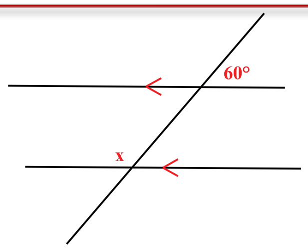
Since $\overline{AB} \parallel \overline{CD}$ and $\overline{EF} \perp \overline{CD}$ then $\overline{EF} \perp \overline{AB}$

$\angle 6x = 90$

$x = 15$

➢ **C**

4.15 Find x



A 60 **B** 120

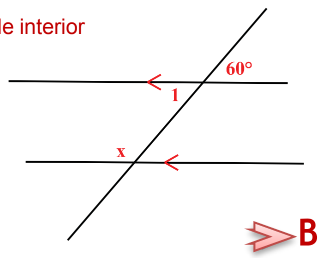
C 180 **D** 100

$\angle 60 = \angle_1$ Vertical Angles

$x + \angle_1 = 180$ Same side interior

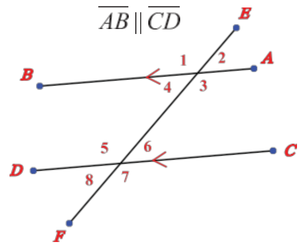
$x + 60 = 180$

$x = 120$



➢ **B**

4.16 One of the following facts is not needed to prove that



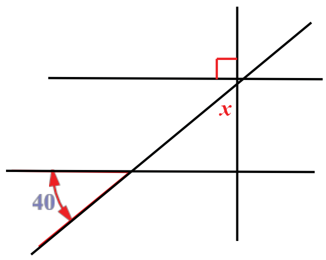
A $\angle_2 = \angle_4$ **B** $\angle_1 = \angle_5$

C $\angle_3 = \angle_7$ **D** $\angle_3 + \angle_6 = 180$

$\angle_2 = \angle_4$ are vertical angles and is not related to $\overline{AB} \parallel \overline{CD}$

➢ **A**

4.17 Find x



A 40 **B** 30

C 50 **D** 60

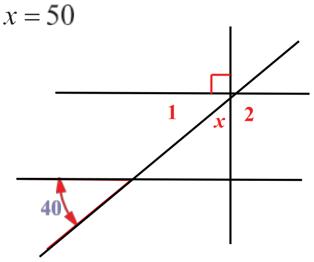
$\angle_2 = 90^\circ$ Vertical Angles

$\angle_1 = 40^\circ$ Corresponding angles

$\angle_1 + \angle_2 + x = 180$ Adjacent on a line

$130 + x = 180$

$x = 50$



➢ **B**