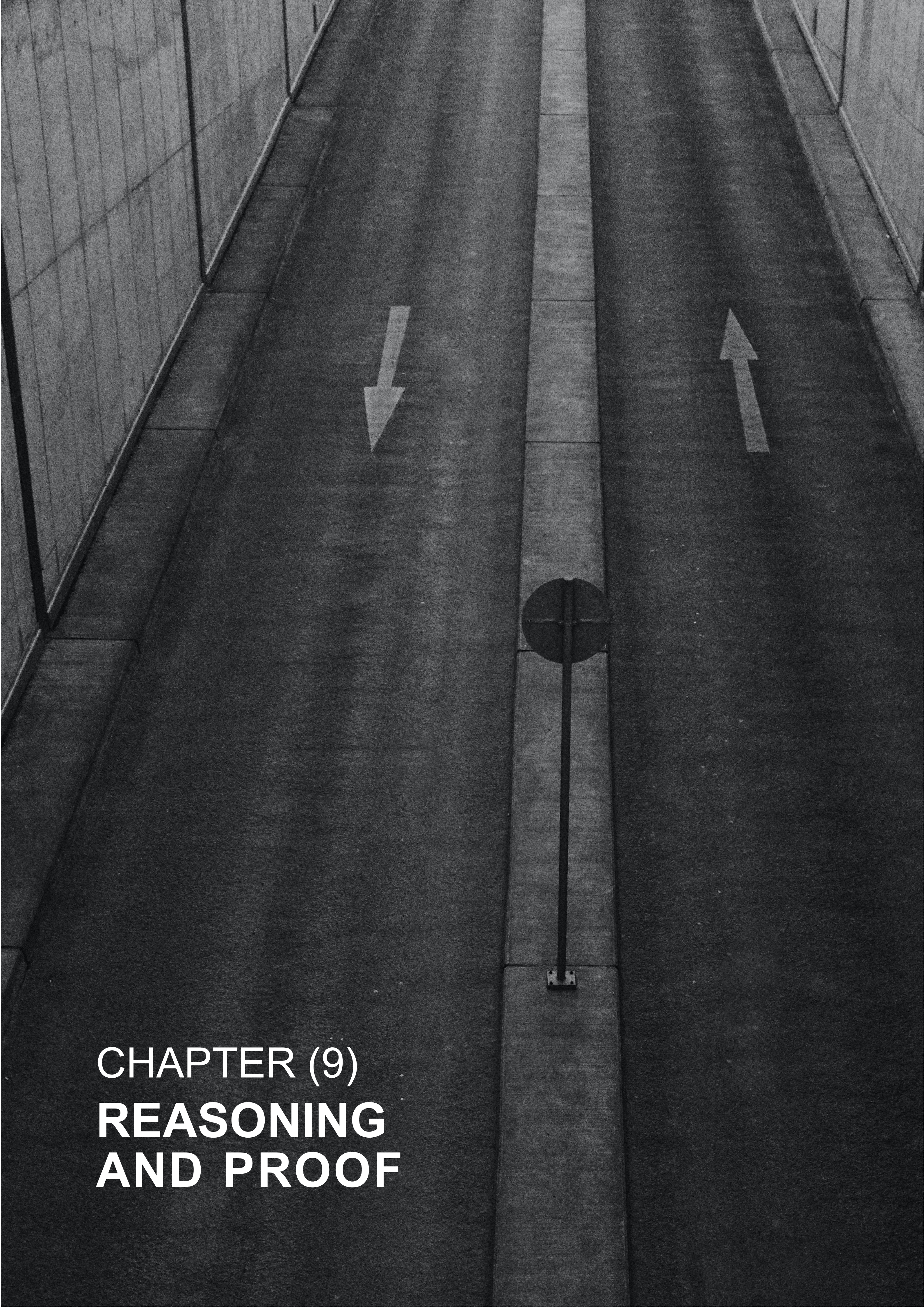


CHAPTER (9)

REASONING AND PROOF

Statements and Negations

- **Statement** is a sentence that could be true *(T)* or false *(F)*. Usually represented by ***p, q ,r, s***
- The **negation of a statement *p*** is the opposite of the statement. The symbol **~*p*** and is read ***'not p'***.

P	*~p*
T	F
F	T

Counter example is an example that shows a conjecture is incorrect

Example

If $x^2 = 36$ then $x = 6$ the counter example is $x = -6$ since $(-6)^2 = 36$

9.1 Find a counter example for the statement: if x is a real number then $x^2 \geq x$

A $x = 2$ **B** $x = -2$

C $x = \frac{3}{2}$ **D** $x = \frac{1}{2}$

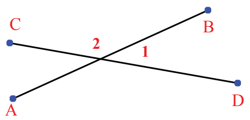
By trying the options:

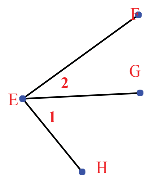
	x	x^2	$x^2 \geq x$
A	2	4	T
B	-2	4	T
C	$\frac{3}{2}$	$\frac{15}{4}$	T
D	$\frac{1}{2}$	$\frac{1}{4}$	F

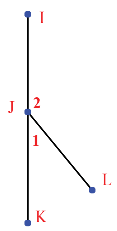
D

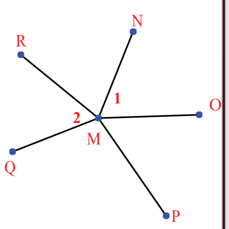
9.2 Find a counter example for the statement

If $\angle 1$ and $\angle 2$ share the same vertex then they are adjacent angles

A 

B 

C 

D 

Angles $\angle 1$ and $\angle 2$ share the same vertex in all the four options but they don't share a common side in option D, therefore it is a counter example.

D

9.3 Determine the false statement

A Parallelogram is a quadrilateral

B The measure of a right angle is $90°$

C The number 804 divides 3

D The sum of two complementary angles is 180

Supplementary angles sum up to $180°$

Complementary angles sum up to $= 90°$ D

9.4 Which statement has a false negation

A $8 - 2 \times 3 = 24$

B The measure of an acute angle is $> 90°$

C $\frac{3}{11} + \frac{5}{11} = \frac{8}{22}$

D 2924 is divisible by 4

The divisibility rule of 4 states that the first two digits should be divisible by $4 \to 24 \div 4 = 6$

"2924 is divisible by 4" is a true statement and its negation is false D

Compound Statement

Conjunction: Connect two or more statements with *''and''* $p \wedge p$, read as *p and q*

Disjunction: correct two or more statements with *'or''* $p \vee p$, read as *p or q*

Conditional is an if then statement $p \longrightarrow q$. Read as: if *p then q or p implies q*

Hypothesis: is the part p

Conclusion: is the part q

p	q	$p \wedge p$	$p \vee p$	$p \to p$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

9.5 If p and q are false, which of the following statement is true

A $p \wedge p$

B $\sim p \to p$

C $p \to \sim q$

D $p \vee p$

Option C:

p	q	$\sim q$	$p \to\sim p$
F	F	T	F→T » T

⋙ **C**

9.6 Find truth values of x and y in following table

p	q	$\sim p \wedge p$
T	T	x
T	F	F
F	T	T
F	F	y

A $x = \text{T}, y = \text{F}$

B $x = \text{T}, y = \text{T}$

C $x = \text{F}, y = \text{F}$

D $x = \text{F}, y = \text{T}$

p	$\sim p$	q	$\sim p \wedge p$
T	F	T	F $\to x$
F	T	F	F $\to y$

⋙ **C**

Let p is the hypothesis: $m\angle A = 115°$

and q is the conclusion: $\angle A$ is obtuse

Statement	How to write it	Example	Symbol	Truth value
Conditional	Use the given hypothesis and conclusion	if $m\angle A = 115°$ then $\angle A$ is obtuse	$p \to q$	T
Converse	Exchange the hypothesis and the conclusion	if $\angle A$ is obtuse then $m\angle A = 115°$	$q \to p$	F
Inverse	Negate both the hypothesis and the conclusion of the conditional	if $m\angle A \neq 115°$ then $\angle A$ is not obtuse	$\sim p \to\sim q$	F
Contrapositive	Negate both the hypothesis and the conclusion of the converse	if A is not obtu then $m\angle A \neq 115°$	$\sim q \to\sim p$	T

9.7 Find the contrapositive of the conditional

Statement:

If the figure is a square, then it is a quadrilateral

A If the figure is a quadrilateral, then it is a square

B If the figure is not a square then it is not a quadrilateral

C If the figure is not a quadrilateral, then it is not a square

D If the figure is not a quadrilateral, then it is a square

Conditional:

Square → quadrilateral

$p \to q$

Contrapositive:

$\sim q \to\sim p$

not quadrilateral → not square ⋙ **C**

9.8 Find the inverse of the conditional statement if $x = 3$ then $x^2 = 9$

A If $x \neq 3$ then $x^2 \neq 9$

B If $x^2 \neq 9$ then $x \neq 3$

C If $x =$

D If $x^2 = 9$ then $x = 3$

Conditional

$x = 3 \to x^2 = 9$

$p \to q$

Inverse

$\sim p \to\sim q$

$x \neq 3 \to x^2 \neq 9$ ⋙ **A**

Indirect Proof

Step 1:
State as a temporary assumption which is the opposite (negation) of what you want to prove.

Step 2:
Show that this assumption leads to a contradiction

Step 3:
Conclude that the temporary assumption must be false and that what you want to prove is true

9.9 Use the indirect proof to show that the following statement is true: If $2x < 18$ then $x < 9$

A $x \leq 9$

B $x \geq 9$

C $x < 9$

D $x > 9$

The conclusion is $x < 9$

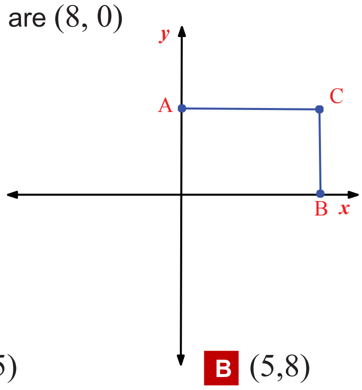
The assumption is the negation is $x \geq 9$ ⋙ **B**

Proofs Using Coordinate Geometry

You will use coordinate with variables to write a coordinate proof

- You can prove geometric relationships using variables coordinates for figures in the coordinate plane.
- All points that lies on the same horizontal line have the same y-coordinate.

9.10 Find the coordinates of the point C if the coordinates of the point A are $(0, 5)$ and the coordinates of the point B are $(8, 0)$



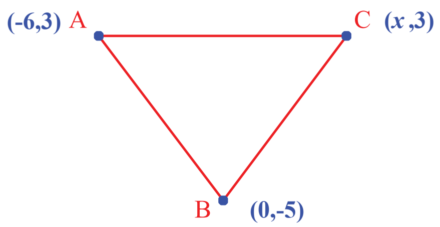
A $(8,5)$ **B** $(5,8)$

C $(5,0)$ **D** $(0,8)$

- Since A and C are on the same horizontal line then they have the same y-coordinate $\rightarrow 5$
- Since B and C are on the same vertical line then they have the x-coordinate$\rightarrow 8$

$C\,(8,5)$ ➢ **A**

9.11 If ΔABC is an isosceles triangle then what is the value of x



A 3 **B** 6

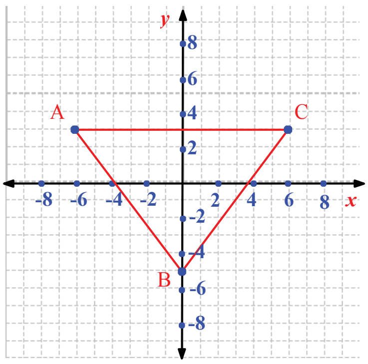
C -6 **D** -3

By graphing the triangle on the coordinate grid then y *axis* is an *axis* of symmetry the point C is reflection of A across

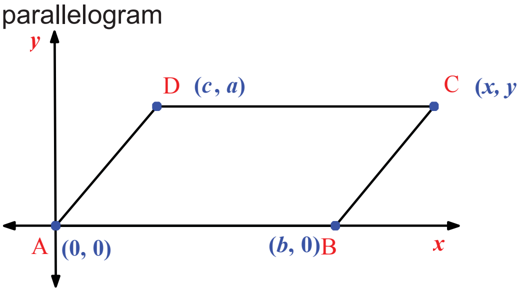
y − axis Reflection

$(a,b) \rightarrow (-a,b)$

$(-b,3) \rightarrow (6,3), x = 6$



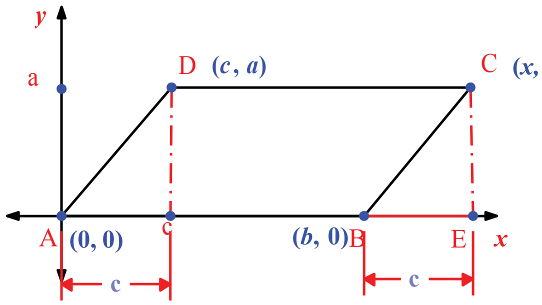
9.12 What are the coordinates of the point C if $ABCD$ is a parallelogram



A (b,c) **B** $(b,-c)$

C $(b+c,a)$ **D** (c,a)

- Since D and C are on the same horizontal line $\rightarrow y = a$



- The x coordinate of point B is b
  The x coordinate of point E is $b + c$

$(b+c,a)$ ➢ **C**