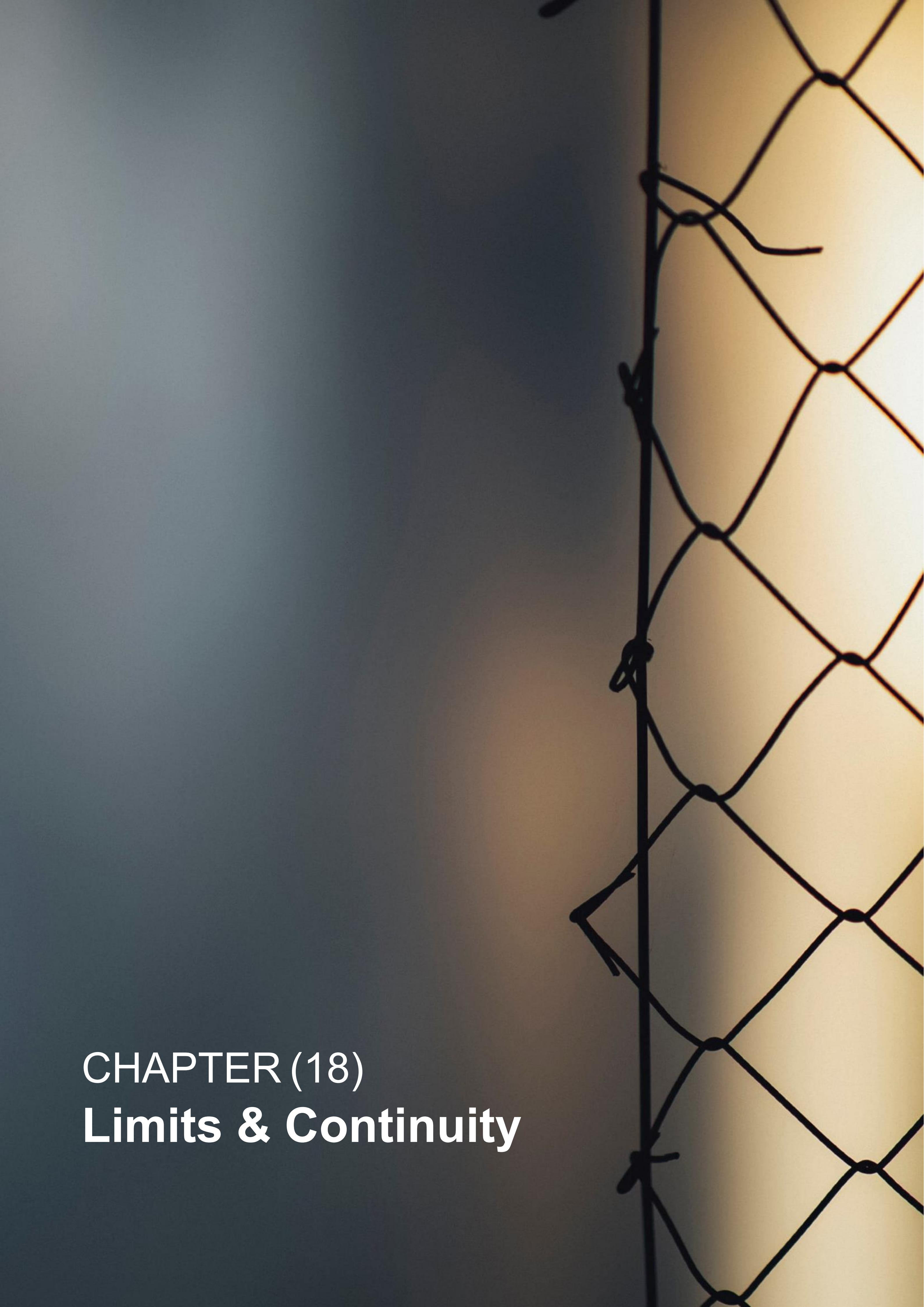
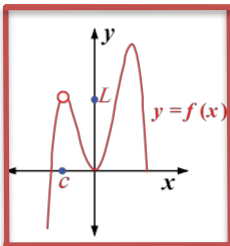
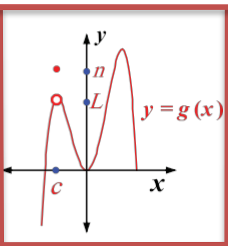
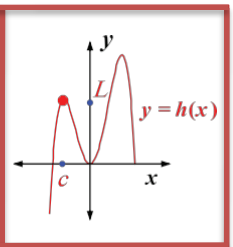


CHAPTER (18)

Limits & Continuity

- The limit of a function $f(x)$ as x approaches c does not depend on the value of the function at point c

		
$\lim_{x\to c} f(x) = L$	$\lim_{x\to c} g(x) = L$	$\lim_{x\to c} h(x) = L$
$f(c)$ is undefined	$g(c) = n$	$h(c) = L$

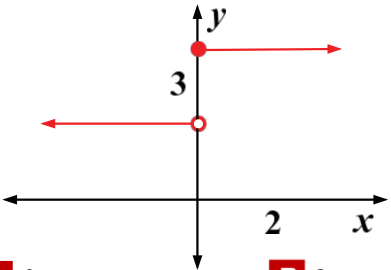
One side limit	Read as
$\lim_{x\to c^-} f(x) = L_1$	The limit of $f(x)$ as x approaches c from left is L_1
$\lim_{x\to c^+} f(x) = L_2$	The limit of $f(x)$ as x approaches c from right is L_2

- The limit of a function $f(x)$ as x approaches c exists if and only if both one-sided limits exist and are equal. That is if

$\lim_{x\to c^-} f(x) = \lim_{x\to c^-} f(x) = L$, then $\lim_{x\to c} f(x) = L \rightarrow$ exist

$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^-} f(x)$, then $\lim_{x\to c} f(x)$ does not exist

1. Use the graph of $f(x)$ to estimate the $\lim_{x\to 0^-} f(x)$



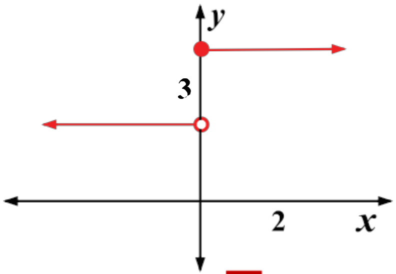
A 0 **B** 2

C 3 **D** does not exist

Because the left-hand limit of $f(x)$ as x approaches $\mathbf{0}$ is $\mathbf{2}$ and the limit does not depend on the value of the function at x [f(0)=4],

then $\lim_{x\to 0^-} f(x) = 2$ ⇒ **B**

2. Use the graph of $f(x)$ to estimate the $\lim_{x\to 2} f(x)$



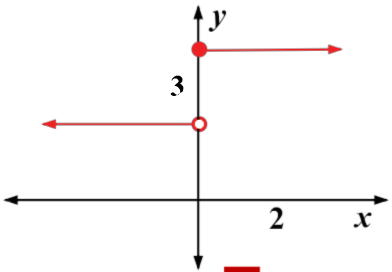
A 4 **B** 2

C 0 **D** does not exist

The left and right-hand limits of $f(x)$ as x approaches $\mathbf{2}$ are the same and equal $\mathbf{4}$,

$\lim_{x\to 2} f(x) = 4$ ⇒ **A**

3. Use the graph of $f(x)$ to estimate the $\lim_{x\to 0} f(x)$



A 0 **B** 2

C -2 **D** does not exist

The graph of $f(x)$ suggests that

$\lim_{x\to 0^-} f(x) = +2$, but $\lim_{x\to 0^+} f(x) = 4$

Since they are not the same $\lim_{x\to 0} f(x)$ does not exist ⇒ **D**

4. If $\lim_{x\to 5^+} f(x) = 4$, $\lim_{x\to 5^-} f(x) = 4$ and $f(5) = 8$, then find $\lim_{x\to 5}$

A 4 **B** 5

C 8 **D** does not exist

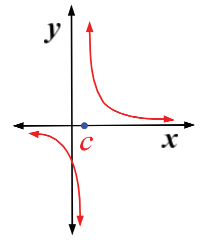
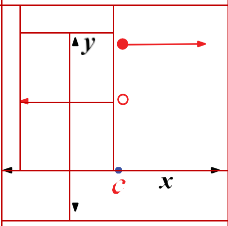
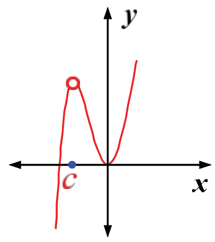
Since $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x) = 4$

and the limit does not depend on $f(5)$

then the $\lim_{x\to 5} f(x) = 4$ ⇒ **A**

Continuity

The graph of a ***continuous function*** has no breaks, holes or gaps. You can trace the graph of a continuous function without lifting your pencil.

Types of discontinuity		
Infinite discontinuity	**Jump discontinuity**	**Removable discontinuity**
The function values increases or decreases indefinitely as x approaches c from the left and right	The limits of the function as x approaches c from the left and right exist but have two distinct values	The function is continuous everywhere except for a hole at $x = c$
		

- To determine continuity of a function $f(x)$ at a point $x = a$, $f(x)$ is continuous at $x = a$ if and only if $f(a) = \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$

5. Find k such that then find $f(x)$ is continuous at $x = 1$

$$f(x) = \begin{cases} 2x^2 + 7 & x \ge 1 \\ kx - 1 & x < 1 \end{cases}$$

A 10 **B** 9

C 2 **D** 7

$f(x)$ is continuous at $x = 1$ if and only if

$$f(1) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$$

$$f(1) = \lim_{x\to 1^+} (2x^2 + 7) = \lim_{x\to 1^-} kx - 1$$

$$2(1) + 7 = k(1) - 1$$

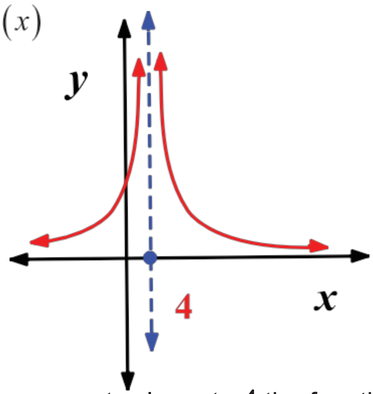
$$9 = k - 1 \rightarrow k = 10$$ > **A**

Limit Fail to Exist

Another way a limit can fail to exist is when the value of $f(x)$ as x approaches c does not approach a fixed finite value. Instead the value of $f(x)$ increases without bound indicated by ∞, or decreases without bound, indicated by $-\infty$

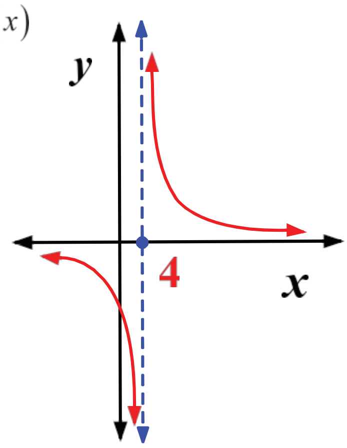
Example

1) Estimate $\lim_{x\to 4} f(x)$



Solution: Because as x gets closer to 4 the function value of the graph increase, both sides tend to ∞, therefore the $\lim_{x\to 4} f(x) = \infty$

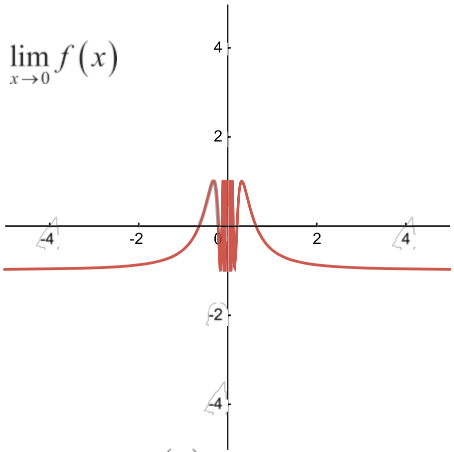
2) Estimate $\lim_{x\to 0} f(x)$



Solution: Because as x gets closer to 4, the function values from the left decrease and the function values from the right increase. In this case we cannot describe the behaviour of $f(x)$ at 4 using single expression

$\rightarrow \lim_{x\to 4} f(x)$ does not exist

3) Estimate $\lim_{x \to 0} f(x)$



Solution: The graph of $f(x)$ suggests that as x gets closer to $\mathbf{0}$, the corresponding function values oscillate between $-\mathbf{1}$ and $\mathbf{1}$ continuously, therefore $\lim_{x \to 0} f(x)$ does not exist

Summary: Why limits at a point does not exist

The limit of $f(x)$ as x approaches c does not exist if:

- $f(x)$ approaches a different value from the left of c than from the right
- $f(x)$ increases or decreases without bound from the left and / or the right of c

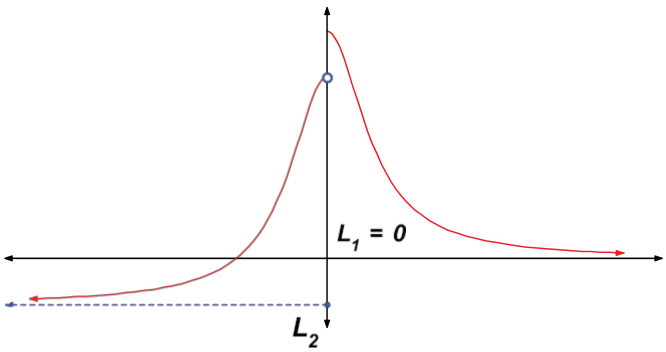
  $f(x)$

Limits at Infinity

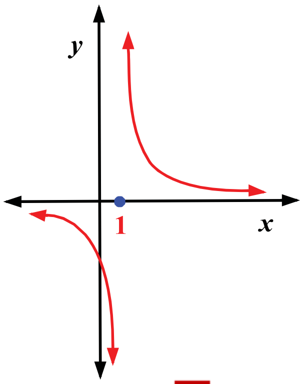
If the value of $f(x)$ approaches a unique number L_1 as x increases, then $\lim_{x \to \infty} f(x) = L_1$

If the value of $f(x)$ approaches a unique number L_2 as x decreases, then $\lim_{x \to -\infty} f(x) = L_2$

$f(x)$ oscillates between two fixed values



6. Use the graph of $f(x)$ to estimate $\lim_{x \to 1} f(x)$

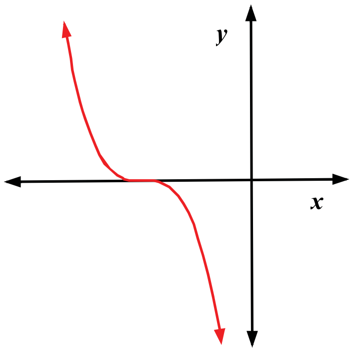


A 0 **B** ∞

C $-\infty$ **D** does not exist

$\lim_{x \to 1^+} f(x) = +\infty \neq \lim_{x \to 1^-} f(x) = -\infty$ ➤ **D**

7. Describe the left end behaviour



A $\lim_{x \to \infty} f(x) = \infty$ **B** $\lim_{x \to -\infty} f(x) = \infty$

C $\lim_{x \to 0} f(x) = 0$ **D** $\lim_{x \to -\infty} f(x) = -\infty$

As x decreases $(x \to -\infty)$,
the values of $f(x)$ increase (∞)

$\to \lim_{x \to -\infty} f(x) = \infty$ ➤ **B**

Evaluating Limits Algebraically

- Limits of constant functions at any point c is the constant value of the function $\lim_{x\to c} k = k$
- The limit of the identity function at any point c is c $\lim_{x\to c} x = c$
- You can use substitution to evaluate limits $\lim_{x\to c} f(x) = f(c)$
- If you evaluate the limit of a rational function and reach the form $\frac{0}{0}$ you should try to simplify the expression algebraically by factoring and dividing out a common factor or multiply by the conjugate

8. Evaluate the limit $\lim_{x\to 4}(x^2 - 6x + 3)$

A 5 B -5
C -8 D -9

$$\to \lim_{x\to 4}(x^2 - 6x + 3) = (4)^2 - 6(4) + 3$$
$$= 16 - 24 + 3$$
$$= -5$$
➢ B

9. Evaluate the limit $\lim_{x\to -2}\frac{4x^3 + 1}{x - 5}$

A $\frac{-31}{7}$ B $\frac{28}{7}$
C $\frac{-28}{7}$ D $\frac{31}{7}$

$$\lim_{x\to -2}\frac{4x^2 + 1}{x - 5} = \frac{4(-2)^3 + 1}{-2 - 5}$$
$$= \frac{31}{7}$$
➢ D

10. Evaluate the limit $\lim_{x\to 7}\frac{\sqrt{2x + 2} - \sqrt{5}}{x - 6}$

A $4 + \sqrt{5}$ B $4 - \sqrt{5}$
C $\sqrt{5}$ D $-\sqrt{5}$

$$\lim_{x\to 7}\frac{2x + 2}{x - 6} = \frac{\sqrt{2(7) + 2} - \sqrt{5}}{7 - 6}$$
$$= \frac{\sqrt{16} - \sqrt{5}}{1}$$
$$= 4 - \sqrt{5}$$
➢ B

11. Evaluate $\lim_{x\to 3}\frac{x^2 - 9}{x - 3}$

A does not exist B 6
C -6 D 0

$$\lim_{x\to 3}\frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{(x-3)}$$
$$= \lim_{x\to 3}(x+3)$$
$$= 3+3$$
$$= 6$$
➢ B

12. Evaluate $\lim_{x\to 0}\frac{(x-4)^2 - 16}{x}$

A does not exist B 8
C -8 D 0

$$\lim_{x\to 0}\frac{(x - 4)^2 - 16}{x} = \lim_{x\to 0}\frac{x^2 - 8x + 16 - 16}{x}$$
$$= \lim_{x\to 0}\frac{x^2 - 8x}{x}$$
$$= \lim_{x\to 0}\frac{x(x - 8)}{x}$$
$$= \lim_{x\to 0} x - 8$$
$$= 0 - 8$$
$$= -8$$
➢ C

13. Evaluate $\lim_{x\to 16}\frac{x - 16}{\sqrt{x} - 4}$

A does not exist B 0
C 8 D -8

Use difference of squares to factor $x - 16$

$$\lim_{x\to 16}\frac{x - 16}{\sqrt{x} - 4} = \lim_{x\to 16}\frac{(\sqrt{x} - 4)(\sqrt{x} + 4)}{(\sqrt{x} - 4)}$$
$$= \lim_{x\to 16}\sqrt{x} + 4$$
$$= \sqrt{16} + 4$$
$$= 8$$
➢ C

14. Evaluate $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

A $\frac{1}{6}$ B $-\frac{1}{6}$

C does not exist D 0

Use difference of squares to factor $x-9$

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)}$$
$$=\frac{1}{\sqrt{x}+3}$$
$$=\frac{1}{\sqrt{9}+3}$$
$$=\frac{1}{6}$$

➤A

Limits of Power Functions at Infinity

For any positive integer ***n***

- $\lim_{x\to\infty} x^n=\infty$
- $\lim_{x\to\infty} ax^n=a\lim_{x\to\infty} x^n$
- $\lim_{x\to-\infty} x^n=\infty$, ***n*** is even
- $\lim_{x\to-\infty} x^n=-\infty$, ***n*** is odd
- $\lim_{x\to\pm\infty}\frac{1}{x}=0$
- $\lim_{x\to\pm\infty}\frac{1}{x^n}=0$
- For $f(x)=\frac{a(x)}{b(x)}=\frac{a_n x^n+.....}{b_m x^m+.....}$
  - $\lim_{x\to\infty} f(x)=0$ if $n<m$
  - $\lim_{x\to\infty} f(x)=\frac{a_n}{a_m}$ if $n=m$
  - $\lim_{x\to\infty} f(x)=\infty$ if $n>m$

15. Evaluate $\lim_{x\to\infty}\left(x^3+2x^2+1\right)$

A 0 B ∞

C $-\infty$ D 1

$\lim_{x\to\infty} x^n=\infty$ ➤B

16. Evaluate $\lim_{x\to\infty}\frac{12x^5-7}{3x^5+2x^2-7x}$

A 1 B 4

C 0 D ∞

$m=n$ $\lim_{x\to\infty} f(x)=\frac{a_n}{a_m}$
$$=\frac{12}{3}$$
$$=4$$
➤B

17. Evaluate $\lim_{x\to\infty}\frac{2x^2+1}{3x-8}$

A 0 B $\frac{2}{3}$

C 1 D ∞

$\lim_{x\to\infty} f(x)=\infty$ if $n>m$ ➤D

18. Evaluate $\lim_{x\to-\infty}\frac{4}{5n^3+2}$

A $\frac{4}{5}$ B 2

C 0 D ∞

$\lim_{x\to-\infty} f(x)=0$ ➤C

19. Evaluate $\lim\limits_{x \to \infty} \dfrac{15x^3 - 12x^2 + 1}{5x^2 + 3x^3 + 4}$

A 3 **B** 5

C 0 **D** ∞

Rewrite the denominator in standard form

$$\lim_{x \to \infty} \frac{15x^3 - 12x^2 + 1}{3x^3 - 5x^2 + 4}$$

$m = n \rightarrow \lim\limits_{x \to \infty} f(x) = \dfrac{a_n}{b_m}$

$= \dfrac{15}{3}$

$= 5$ **B**

20. Let $\lim\limits_{x \to \infty} \dfrac{Bx}{2 + |x|} = 3,$ then find $\boldsymbol{B}$

A $\dfrac{1}{3}$ **B** 3

C -3 **D** 0

Rewrite the denominator in standard form

$|x| = x$ as x approaches $+\infty$

$$\lim_{x \to \infty} \frac{Bx}{2 + |x|} = \lim_{x \to \infty} \frac{Bx}{2 + x}$$

$n = m \quad \lim\limits_{x \to \infty} f(x) = \dfrac{a_n}{b_m} = \dfrac{B}{1} = 3$

$\rightarrow B = 3$ **B**