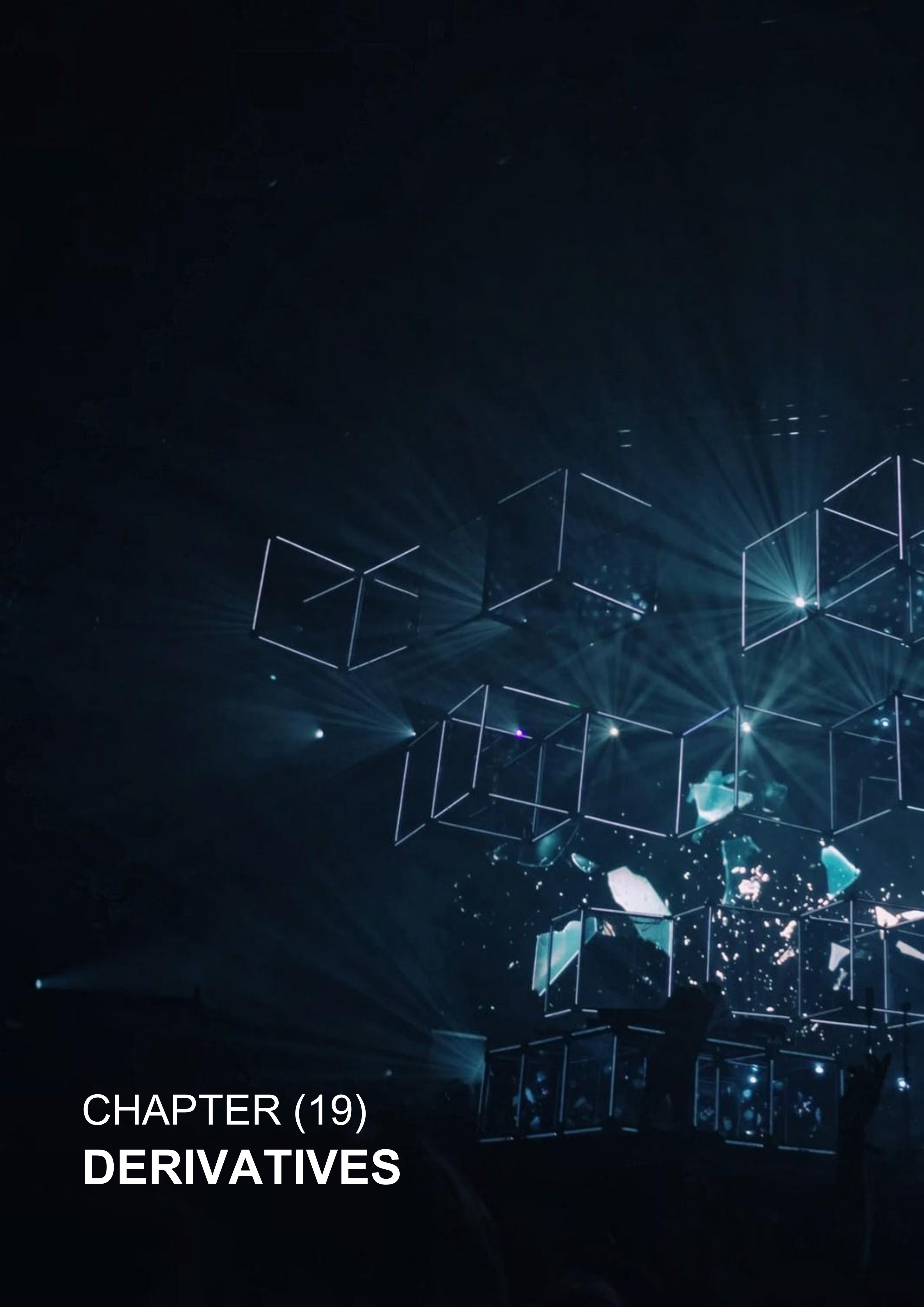
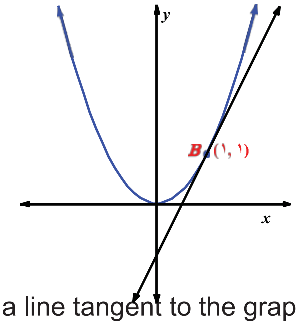


CHAPTER (19)

DERIVATIVES

- Tangent line is a line that just touches a curve at only one point



- The slope of a line tangent to the graph of a function is called the derivative of a function.
- The process of finding a derivative is called differetion and the result is called differential equation, and is denoted by $f'(x), y' \ or \ \frac{dy}{dx}$

Rules for Derivatives

Rule	$f(x)$	$f'(x)$	Example
Constant	$f(x)=c$	$f'(x)=0$	$f(x)=5$ $f'(x)=0$
Power	$f(x)=x^n$	$f'(x)=nx^{n-1}$	$f(x)=x^4$ $f'(x)=4x^3$
Constant Multiple	$f(x)=cx^n$	$f'(x)=cnx^{n-1}$	$f(x)=2x^4$ $f'(x)=4\cdot 2x^3$ $=8x^3$
Sum or difference	$f(x)=g(x)\pm h(x)$	$f'(x)=g'(x)\pm h'(x)$	$f(x)=x^3+5x^2-4$ $f'(x)=3x^2+10x-0$
nth derivative	$f(x)=x^n$	$f'(x)=nx^{n-1}$ $f''(x)=n(n-1)x^{n-2}$	$f(x)=x^4$ $f'(x)=4x^3$ $f''(x)=12x^2$ $f'''(x)=24x$ $f^{(4)}=24$

19.1 Find $f'(x)$ if $f(x)=x+1$

A 0　　B 1

C 2　　D -1

$f(x)=x+1$

$f'(x)=1+0$

$=1$

➢B

19.2 Find $\frac{dy}{dx}$ if $y=4x^{\frac{5}{4}}+8x^{\frac{1}{2}}+4$

A $5x^{\frac{1}{4}}+4x^{\frac{1}{2}}$　　B $5\sqrt[4]{x}+\frac{4\sqrt{x}}{x}$

C $5\sqrt[4]{x}+\frac{4}{x}$　　D $5x^{\frac{9}{4}}+4x^{\frac{1}{2}}$

$$y=4x^{\frac{5}{4}}+8x^{\frac{1}{2}}+4$$

$$\frac{dy}{dx}=\frac{5}{4}\cdot 4x^{\frac{5}{4}-\frac{4}{4}}+\frac{1}{2}\cdot 8x^{\frac{1}{2}-\frac{2}{2}}+0$$

$$=5x^{\frac{1}{4}}+4x^{\frac{-1}{2}}$$

$$=5\sqrt[4]{x}+\frac{4}{\sqrt{x}}$$

$$=5\sqrt[4]{x}+\frac{4\sqrt{x}}{x}$$

➢B

19.3 Find an equation for the line tangent to the curve $y=3x^4+2x^3+5x-4$

$4x^3+3x+5$　　B $12x^4+6x^3+5x-4$

C $12x^3+6x^2+5$　　D $12x^4+6x^3+5$

To find an equation for the line tangent we differentiate ***y***

$y=3x^4+2x^3+5x-4$

$y'=12x^3+6x^2+5$

➢C

19.4 Find $f'(x)$ if $f(x)=\sqrt[9]{x^5}$

A $\frac{5}{9}x^{\frac{5}{9}}$　　B $\frac{5}{9}x^{\frac{4}{9}}$

C $\frac{5}{9}\sqrt[9]{x^4}$　　D $\frac{5}{9\sqrt[9]{x^4}}$

$$f(x)=\sqrt[9]{x^5}$$

$$=x^{\frac{5}{9}}$$

$$f'(x)=\frac{5}{9}x^{\frac{5}{9}-\frac{9}{9}}$$

$$=\frac{5}{9}x^{\frac{-4}{9}}$$

$$=\frac{5}{9}\cdot\frac{1}{\sqrt[9]{x^4}}=\frac{5}{9\sqrt[9]{x^4}}$$

➢D

19.5 Find the slope for the line tangent to the curve $f(x)=10x^2-3x+4$ at the point $(1, 3)$

A 20　　B 17

C 23　　D 11

Step 1: Differentiate

$f(x)=10x^2-3x+4$

$f'(x)=20x-3$

Step 2: Find the slope at the point

$(1, 3) \ \rightarrow \ x=1$

$f'(1)=20(1)-3$

$=17$

➢B

19.6 Find the second derivative of the function $f(x) = 3x^4 - 2x^3 + x + 4$

A $12x^3 - 6x^2 + 1$ **B** $36x^2 - 12x$

C $12x^3 - 6x^2$ **D** $36x^2 - 12x + 1$

$f(x) = 3x^4 - 2x^3 + x + 4$

$f'(x) = 12x^3 - 6x^2 + 1$

$f''(x) = 36x^2 - 12x$

➢**B**

19.7 Find the seventh derivative of $f(x) = 3x^6 + 4x^5 + 2x^3 + \frac{1}{2}x^2 - 7x + 1$

A 18 **B** 0

C 1 **D** 2

Since the required derivative is the 7th which is greater than the degree of the polynomial, therefore $f^{(7)}(x) = 0$

➢**B**

Product and Quotient Rules for Derivatives

Rule	$h(x)$	$h'(x)$	Example
Product	$h(x)=f(x)\,g(x)$	$f'(x)\,g(x)+f(x)\,g'(x)$ (first)'(second)+(first)(second)	$h(x)=(x^3+4)(2x^2)$ $f=x^3+4$, $f'=3x^2$ $g=2x^2$, $g'=4x$ $3x^2(2x^2)+4x(x^3+4)$
Quotient	$h(x)=\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)\,g(x)-f(x)\,g'(x)}{[g(x)]^2}$ $\dfrac{(n)'(d)-(n)(d)'}{d^2}$ n = numirator d = derominater	$h(x)\,\dfrac{x^3+4}{2x^2}$ $f=x^3+4$, $f'=3x^2$ $g=2x^2$, $g'=4x$ $\dfrac{3x^2(2x^2)-4x(x^3+4)}{(2x^2)^2}$

19.8 Find $f(x) = 3x^2 - kx + 5$ **if** $f'(1) = 2$

A 3 **B** 4

C -8 **D** -4

$f(x) = 3x^2 - kx + 5$

$f'(x) = 6x - k$

$f'(1) = 6(1) - k$

$2 = 6 - k$

$k = 4$

➢**B**

19.9 Find $f'(x)$ **if** $f(x) = (x^2 - 3)(4x + 1)$

A $12x^2 + 2x$ **B** $12x^2 + 2x - 12$

C $2x^2 - 2$ **D** $x^2 + 2x + 1$

$f(x) = (x^2 - 3)(4x + 1)$

$f = x^2 - 3$ $\quad f' = 2x$

$g = 4x + 1$ $\quad g' = 4$

$f'(x)\,g(x) + f(x)\,g'(x)$

$2x(4x + 1) + 4(x^2 - 3)$

$8x^2 + 2x + 4x^2 - 12$

$12x^2 + 2x - 12$

➢**B**

19.10 Find $f'(x)$ **if** $f(x) = \frac{8}{x + 6}$

A $\frac{-8}{(x+6)^2}$ **B** $\frac{8}{(x+6)^2}$

C $\frac{-8}{(x+6)}$ **D** $\frac{8}{(x+6)}$

$$\frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$= \frac{0\,(x+6) - 1\cdot 8}{(x+6)^2}$$

$$= \frac{-8}{(x+6)^2}$$

➢**A**

Velocity and Acceleration

Let the speed of an object given by the equation $s = f(t)$ then the velocity $v = f'(t)$ and acceleration is given by $a = f''(t)$

19.11 If the speed of an object at any time is given by $s(t) = 16t - 5t^2 + 4$, then find the equation of its instantaneous velocity.

A $16 - 10t$ **B** $16t - 5$

C $16 - 5t$ **D** $16 - t$

$s(t) = 16t - 5t^2 + 4$

$v = s'(t) = 16 - 10t$

➢**A**

19.12 If the speed of an object at any time is given by $s(t) = 24t - t^2$, find the instantaneous velocity after 3 seconds

A 81 B 26

C 30 D 6

$$s(t) = 24t + t^2$$
$$v = s'(t) = 24 + 2t$$
$$s'(3) = 24 + 2(3) = 30$$

➢C

19.13 Find the acceleration of an object if its speed at any time is given by $s(t) = 2x^3 + 4 + x^2$

A $6x^2 + 2x$ B $6x^2 + 2x + 4$

C $12x^2 + 2x$ D $12x + 2$

$$s(t) = 2x^3 + x^2 + 4$$
$$v \to s'(t) = 6x^2 + 2x$$
$$a \to s''(t) = 12x + 2$$

➢D

19.14 If $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$ then find $\frac{d}{dx}(\sin x \cos x)$

A $\sin^2 x - \cos^2 x$ B $\sin 2x$

C $\cos^2 x + \sin^2 + x$ D $\cos 2x$

$f(x) = \sin x$ and $g(x) = \cos x$

$$\frac{d}{dx}[f(x)\, g(x)] = f'(x)\, g(x) + f(x)\, g'(x)$$

$f = \sin x \qquad f' = \cos x$

$g = \cos x \qquad g' = -\sin x$

$\cos x \cos x + (-\sin x)(\sin x)$

$\cos^2 x - \sin^2 x$

➢D

19.15 Find the acceleration of an object after 2 seconds if its speed at any time is given by $s(t) = x^2\left(x^3 + 1\right)$

A 158 B 36

C 84 D 162

$$s(t) = x^2\left(x^3 + 1\right) = x^5 + x^2$$
$$v \to s'(t) = 5x^4 + 2x$$
$$a \to s''(t) = 20x^3 + 2$$
$$s''(2) = 20(2)^3 + 2 = 160 + 2 = 162$$

➢D

Extreme Values of Functions

- Extreme values are any maximum or minimum value on a given interval and where these values are located if it does.
- Critical point is a point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist.
- The extreme values found at the end points and at critical points.

19.16 The critical point of the function $f(x) = 4x - 3x^2 + 5$ is at $x =$ .....

A $\frac{3}{2}$ B $\frac{-2}{3}$

C $\frac{2}{3}$ D $\frac{-3}{2}$

$$f(x) = 4x - 3x^2 + 5$$
$$f'(x) = 4 - 6x$$

To find the critical point let $f'(x) = 0$

$$4 - 6x = 0$$
$$6x = 4$$
$$x = \frac{4}{6}$$
$$x = \frac{2}{3}$$

➢C

Example:

Find the maximum value of the function $f(x)$ in the interval $[0, 3]$ where $f(x) = \frac{x^3}{3} - 3x^2 + 8x - 4$

Step 1: Find the critical points

$$f(x) = \frac{x^3}{3} - 3x^2 + 8x - 4$$
$$f'(x) = x^2 - 6x + 8$$
$$\text{let } f(x) = 0$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$
$$\rightarrow x = 4 \text{ or } x = 2$$

Step 2: Compare x values to the interval

$x = 4 \notin [0, 3] \rightarrow$ rejected

$x = 2 \in [0, 3] \rightarrow$ accepted

Step 3: Find the values of the endpoints and the critical points: $0, 2, 3$

$$f(0) = \frac{0^3}{3} - 3(0)^2 + 8(0) - 4$$
$$= -4$$
$$f(2) = \frac{2^3}{3} - 3(2)^2 + 8(2) - 4$$
$$= \frac{8}{3}$$
$$f(3) = \frac{3^3}{2} - 3(3)^2 + 8(3) - 4$$
$$= 6.5$$

The maximum value of the function $f(x)$ in the interval $[0, 3]$ is 6.5

19.17 The maximum value of the function $f(x) = 3x^3 - 16x$ in the internal $[-5, 0]$ is at

A $x = \pm\frac{4}{3}$

B $x = \frac{-4}{3}$

C $x = \frac{4}{3}$

D $x = -\frac{3}{4}$

Step 1: Find the critical point

$$f(x) = 3x^3 - 16x$$
$$f'(x) = 9x^2 - 16$$
$$\text{let } f'(x) = 0$$
$$9x^2 - 16 = 0$$
$$(3x-4)(3x+4) = 0$$
$$x = \pm\frac{4}{3}$$

A

Step 2: Compare x values to the interval

$x = \frac{4}{3} \notin [-5, 0]$ rejected

$x = \frac{-4}{3} \in [-5, 0]$ accepted

B