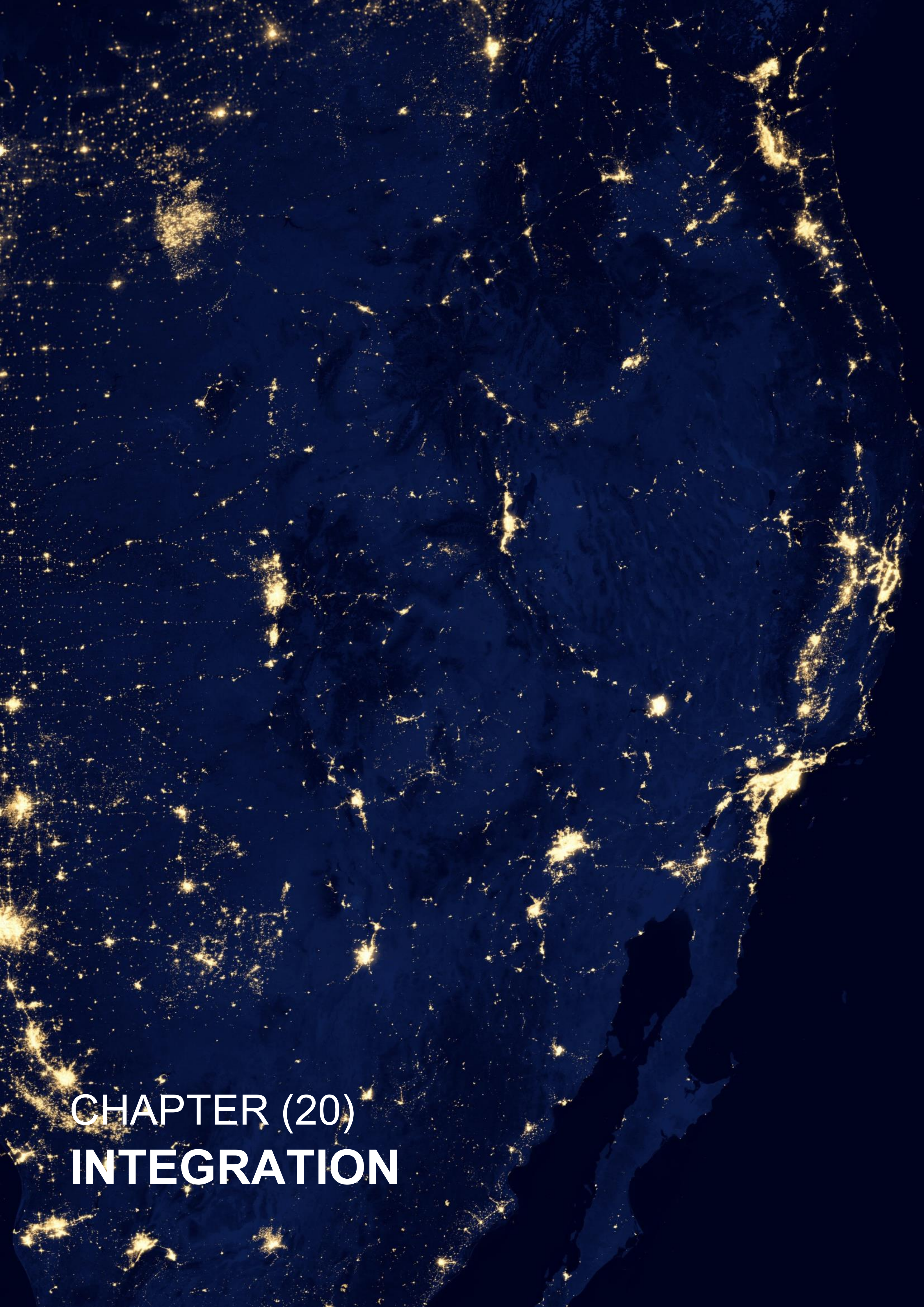


CHAPTER (20)

INTEGRATION

- The antiderivative of $f(x)$ is $F(x)$

$\int f(x)\,dx = F(x) + c$

Properties of Indefinite Integrals

For any constant k

$\int k\,dx = kx + C$

$\int k f(x)\,dx = k\int f(x)\,dx$

Sum and difference

$\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$

Power Formula

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \qquad n \neq -1$

1. **Find** $\int 12x^5\,dx$

A $3x^4 + C$ **B** $2x^6 + C$

C $60x^4 + C$ **D** $\frac{12}{5}x^5 + C$

$$\int 12x^5\,dx = \frac{12x^{5+1}}{5+1} + C = \frac{12x^6}{6} + C = 2x^6 + C$$

B

2. **Find** $\int \frac{7}{4}\sqrt[4]{x^3}\,dx$

A $x^{\frac{4}{7}} + C$ **B** $\frac{7}{4}x^{\frac{7}{4}} + C$

C $\sqrt[4]{x^7} + C$ **D** $\frac{7}{4}\sqrt[4]{x^7} + C$

$$\int \frac{7}{4}\sqrt[4]{x^3}\,dx = \frac{7}{4}\int x^{\frac{3}{4}}\,dx$$

$$= \frac{7}{4}\,\frac{x^{\frac{3}{4}+\frac{4}{4}}}{\frac{7}{4}} + C = x^{\frac{7}{4}} + C = \sqrt[4]{x^7} + C$$

C

3. **Evaluate** $\int (6x^5 + \frac{8}{x^5} + 4)\,dx$

A $x^6 + \frac{8x^5}{5} + 4x + C$ **B** $x^6 - \frac{2}{x^4} + 4 + C$

C $x^6 - \frac{2}{x^4} + 4x + C$ **D** $x^6 + \frac{8x^5}{5} + 4 + C$

$$\int (6x^5 + \frac{8}{x^5} + 4)\,dx = \int (6x^5 + 8x^{-5} + 4)\,dx$$

$$= \frac{6x^{5+1}}{5+1} + \frac{8x^{-5+1}}{-5+1} + 4x + C$$

$$= \frac{6x^6}{6} + \frac{8x^{-4}}{-4} + 4x + C$$

$$= x^6 - 2x^{-4} + 4x + C$$

$$= x^6 - \frac{2}{x^4} + 4x + C$$

C

4. **Find the antiderivative of** $f(x) = 4x^3 + 2x$

A $12x^2 + 2 + C$ **B** $x^4 + x^2 + C$

C $4x^4 + 2x^2 + C$ **D** $4x^3 + 2 + C$

$$\int (4x^3 + 2x)\,dx = \frac{4x^{3+1}}{4} + \frac{2x^{1+1}}{2} + C = x^4 + x^2 + C$$

B

Fundamental Theorem of Calculus

If f is continuous at every point of $[a, b]$ and if F is the antiderivative of f on $[a, b]$ then

Properties of definite Integrals

$\int_a^b f(x)dx = F(b) - F(a)$

$\int_a^a f(x)dx = 0$

$\int_a^b c\,dx = c(b - a)$, c is constant

5. **Evaluate** $\int_2^3 (6x + 1)\,dx$

A 44 **B** 16

C 10 **D** 24

$$\int_2^3 6x + 1 = \left.\frac{6x^{1+1}}{1+1} + x\right|_2^3 = \left.\frac{6x^2}{2} + x\right|_2^3 = 3x^2 + x\Big|_2^3$$

$$= \left(3(3)^2 + 3\right) - \left(3(2)^2 + 2\right)$$

$$= (27 + 3) - (12 + 2)$$

$$= 30 - 14$$

$$= 16$$

B

Example Find the value of k if $\int_0^k (2x+4)\,dx = 5$

$$\int_0^k (2x+4)\,dx = \left(2\frac{x^2}{2} + 4x\right)_0^k = 5$$
$$= \left(x^2+4x\right)_0^k = 5$$
$$= \left[k^2 + 4(k)\right] - \left[(0)^2 + 4(0)\right] = 5$$
$$= k^2 + 4k = 5$$
$$= k^2 + 4k - 5 = 0$$
$$= (k+5)(k-1) = 0$$
$$k = 1 \quad or \quad k = -5$$

6. **Find *k* if** $\int_0^2 (x+k)\,dx = -20$

A 11 **B** -11

C 9 **D** -9

$$\int_0^2 (x+k)\,dx = \frac{x^2}{2} + kx\Big|_0^2 = -20$$
$$\left[\frac{4}{2} + k(2)\right] - \left[\frac{0}{2} + k(0)\right] = -20$$
$$= 2 + 2k = -20$$
$$2k = -22$$
$$k = -11$$

➢B

7. **Find the value of** $\int_1^7 \frac{x^2}{x^2-9}dx - \int_1^7 \frac{9}{x^2-9}dx + \int_1^7 \frac{1}{3}dx$

A 2 **B** 4

C 6 **D** 8

$$\int_1^7 \frac{x^2}{x^2-9}dx - \int_1^7 \frac{9}{x^2-9}dx + \int_1^7 \frac{1}{3}dx$$
$$= \int_1^7 \left(\frac{x^2}{x^2-9} - \frac{9}{x^2-9} + \frac{1}{3}\right)dx$$
$$= \int_1^7 \left(\frac{x^2-9}{x^2-9} + \frac{1}{3}\right)dx$$
$$= \int_1^7 \left(1 + \frac{1}{3}\right)dx$$
$$= \int_1^7 \frac{4}{3}dx$$
$$= \frac{4}{3}(7-1)$$
$$= 8$$

➢D

8. **Evaluate** $\int_0^2 \sqrt{x^2+6x+9}\;dx$

A 2 **B** 6

C 8 **D** 10

$$\int_0^2 \sqrt{x^2+6x+9}\;dx = \int_0^2 \sqrt{(x+3)^2}\;dx$$
$$= \int_0^2 (x+3)\,dx$$
$$= \frac{x^2}{2} + 3x\Big|_0^2$$
$$= \left(\frac{(2)^2}{2} + 3(2)\right) - 0$$
$$= 2 + 6$$
$$= 8$$

➢C

9. **Find the value of *k* if** $\int_0^2 kx\,dx = 6$

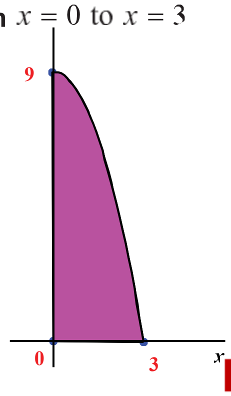
A 3 **B** 6

C 4 **D** 8

$$\int_0^2 kx\,dx = 6 \;\rightarrow\; k\int_0^2 x\,dx = 6$$
$$k\frac{x^2}{2}\Big|_0^2 = 6$$
$$k\left[\frac{2^2}{2} - \frac{0^2}{2}\right] = 6$$
$$k(2-0) = 6$$
$$2k = 6$$
$$k = 3$$

➢A

10. **Find the area under the curve of** $f(x) = 9 - x^2$ **from** $x = 0$ to $x = 3$



A 9 **B** 12

C 6 **D** 18

$$Area = \int_0^3 (9-x^2)\,dx = 9x - \frac{x^3}{3}\Big|_0^3$$
$$= \left(9\cdot 3 - \frac{27}{3}\right)$$
$$= 27 - 9 = 18$$

➢D