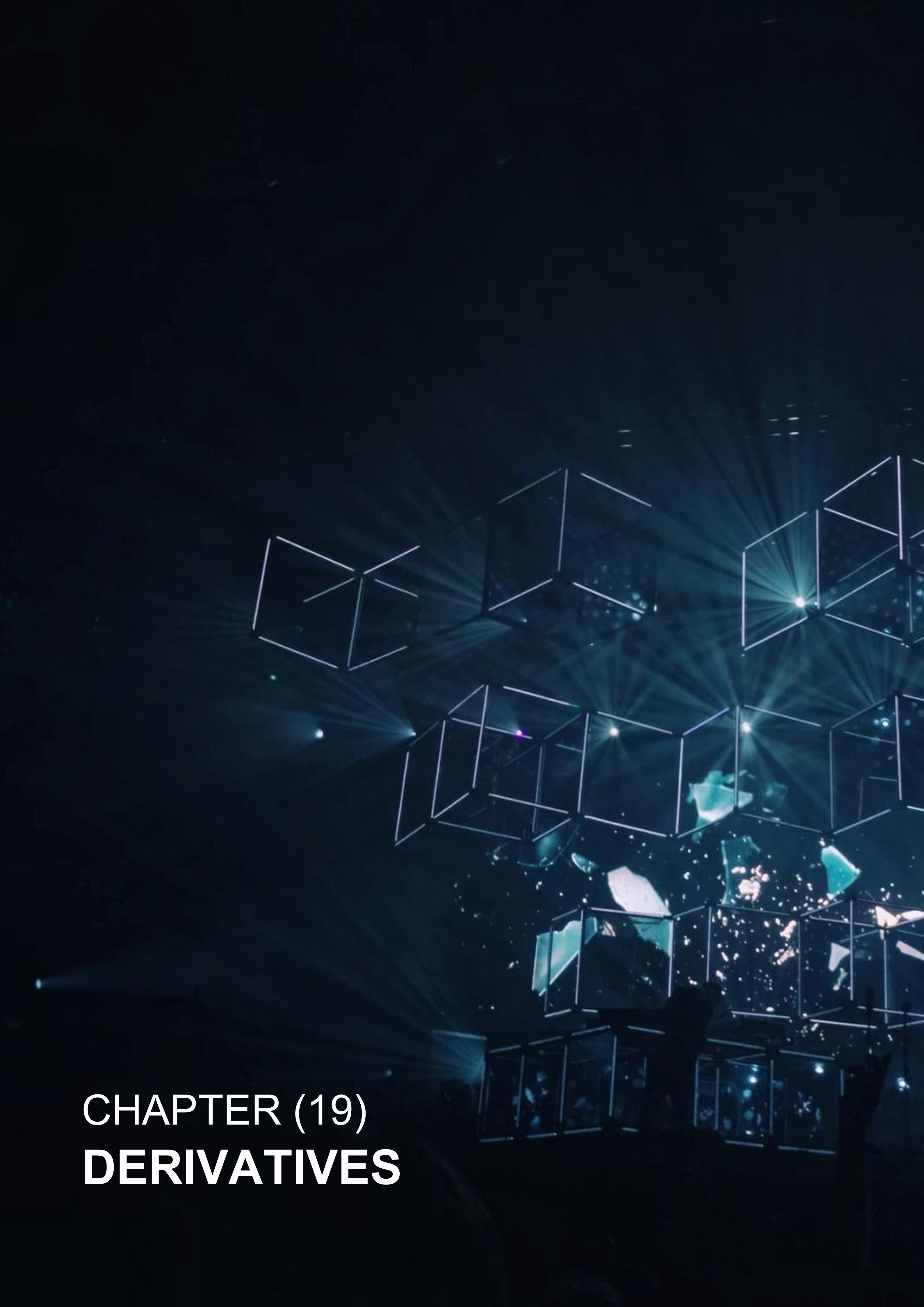
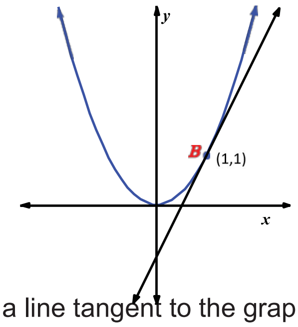


CHAPTER (19)
DERIVATIVES

- Tangent line is a line that just touches a curve at only one point



- The slope of a line tangent to the graph of a function is called the derivative of a function.
- The process of finding a derivative is called differetion and the result is called differential equation, and is denoted by $f'(x), y'$ or $\frac{dy}{dx}$

Rules for Derivatives

Rule	$f(x)$	$f'(x)$	Example
Constant	$f(x)=c$	$f'(x)=0$	$f(x)=5$ $f'(x)=0$
Power	$f(x)=x^n$	$f'(x)=nx^{n-1}$	$f(x)=x^4$ $f'(x)=4x^3$
Constant Multiple	$f(x)=cx^n$	$f'(x)=cnx^{n-1}$	$f(x)=2x^4$ $f'(x)=4\cdot 2x^3$ $=8x^3$
Sum or difference	$f(x)=g(x)\pm h(x)$	$f'(x)=g'(x)\pm h'(x)$	$f(x)=x^3+5x^2-4$ $f'(x)=3x^2+10x-0$
nth derivative	$f(x)=x^n$	$f'(x)=nx^{n-1}$ $f''(x)=n(n-1)x^{n-2}$	$f(x)=x^4$ $f'(x)=4x^3$ $f''(x)=12x^2$ $f'''(x)=24x$ $f^{(4)}=24$

1. Find if $f'(x)$ $f(x)=x+1$

A 0 B 1

C 2 D -1

$$f(x)=x+1$$
$$f'(x)=1+0$$
$$=1$$

>B

2. Find $\frac{dy}{dx}$ if $y=4x^{\frac{5}{4}}+8x^{\frac{1}{2}}+4$

A $5x^{\frac{1}{4}}+4x^{\frac{1}{2}}$ B $5\sqrt[4]{x}+\frac{4\sqrt{x}}{x}$

C $5\sqrt[4]{x}+\frac{4}{x}$ D $5x^{\frac{9}{4}}+4x^{\frac{1}{2}}$

$$y=4x^{\frac{5}{4}}+8x^{\frac{1}{2}}+4$$
$$\frac{dy}{dx}=\frac{5}{4}\cdot 4x^{\frac{5}{4}-\frac{4}{4}}+\frac{1}{2}\cdot 8x^{\frac{1}{2}-\frac{2}{2}}+0$$
$$=5x^{\frac{1}{4}}+4x^{\frac{-1}{2}}$$
$$=5\sqrt[4]{x}+\frac{4}{\sqrt{x}}$$
$$=5\sqrt[4]{x}+\frac{4\sqrt{x}}{x}$$

>B

3. Find an equation for the line tangent to the curve $y=3x^4+2x^3+5x-4$

$4x^3+3x+5$ B $12x^4+6x^3+5x-4$

C $12x^3+6x^2+5$ D $12x^4+6x^3+5$

To find an equation for the line tangent we differentiate y

$$y=3x^4+2x^3+5x-4$$
$$y'=12x^3+6x^2+5$$

>C

4. Find $f'(x)$ if $f(x)=\sqrt[9]{x^5}$

A $\frac{5}{9}x^{\frac{5}{9}}$ B $\frac{5}{9}x^{\frac{4}{9}}$

C $\frac{5}{9}\sqrt[9]{x^4}$ D $\frac{5}{9\sqrt[9]{x^4}}$

$$f(x)=\sqrt[9]{x^5}$$
$$=x^{\frac{5}{9}}$$
$$f'(x)=\frac{5}{9}x^{\frac{5}{9}-\frac{9}{9}}$$
$$=\frac{5}{9}x^{\frac{-4}{9}}$$
$$=\frac{5}{9}\cdot\frac{1}{\sqrt[9]{x^4}}=\frac{5}{9\sqrt[9]{x^4}}$$

>D

5. Find the slope for the line tangent to the curve $f(x)=10x^2-3x+4$ at the point $(1, 3)$

A 20 B 17

C 23 D 11

Step 1: Differentiate

$$f(x)=10x^2-3x+4$$
$$f'(x)=20x-3$$

Step 2: Find the slope at the point

$$(1,3)\ \rightarrow\ x=1$$
$$f'(1)=20(1)-3$$
$$=17$$

>B

6. **Find the second derivative of the function** $f(x) = 3x^4 - 2x^3 + x + 4$

A $12x^3 - 6x^2 + 1$ **B** $36x^2 - 12x$

C $12x^3 - 6x^2$ **D** $36x^2 - 12x + 1$

$f(x) = 3x^4 - 2x^3 + x + 4$

$f'(x) = 12x^3 - 6x^2 + 1$

$f''(x) = 36x^2 - 12x$

➢B

7. **Find the seventh derivative of** $f(x) = 3x^6 + 4x^5 + 2x^3 + \frac{1}{2}x^2 - 7x + 1$

A 18 **B** 0

C 1 **D** 2

Since the required derivative is the 7th which is greater than the degree of the polynomial, therefore $f^{(7)}(x) = 0$

➢B

Product and Quotient Rules for Derivatives

Rule	$h(x)$	$h'(x)$	Example
Product	$h(x)=f(x)\,g(x)$	$f'(x)\,g(x)+f(x)\,g'(x)$ (first)'(second)+(first)(second)	$h(x)=(x^3+4)(2x^2)$ $f=x^3+4$, $f'=3x^2$ $g=2x^2$, $g'=4x$ $3x^2(2x^2)+4x(x^3+4)$
Quotient	$h(x)=\frac{f(x)}{g(x)}$	$\frac{f'(x)\,g(x)-f(x)\,g'(x)}{[g(x)]^2}$ $\frac{(n)'(d)-(n)(d)'}{d^2}$ n = numirator d = derominater	$h(x)\,\frac{x^3+4}{2x^2}$ $f=x^3+4$, $f'=3x^2$ $g=2x^2$, $g'=4x$ $\frac{3x^2(2x^2)-4x(x^3+4)}{(2x^2)^2}$

8. **Find** $f(x) = 3x^2 - kx + 5$ **if** $f'(1) = 2$

A 3 **B** 4

C -8 **D** -4

$f(x) = 3x^2 - kx + 5$

$f'(x) = 6x - k$

$f'(1) = 6(1) - k$

$2 = 6 - k$

$k = 4$

➢B

9. **Find** $f'(x)$ **if** $f(x) = (x^2 - 3)(4x + 1)$

A $12x^2 + 2x$ **B** $12x^2 + 2x - 12$

C $2x^2 - 2$ **D** $x^2 + 2x + 1$

$f(x) = (x^2 - 3)(4x + 1)$

$f = x^2 - 3$ $\quad f' = 2x$

$g = 4x + 1$ $\quad g' = 4$

$f'(x)\,g(x) + f(x)\,g'(x)$

$2x(4x + 1) + 4(x^2 - 3)$

$8x^2 + 2x + 4x^2 - 12$

$12x^2 + 2x - 12$

➢B

10. **Find** $f'(x)$ **if** $f(x) = \frac{8}{x+6}$

A $\frac{-8}{(x+6)^2}$ **B** $\frac{8}{(x+6)^2}$

C $\frac{-8}{(x+6)}$ **D** $\frac{8}{(x+6)}$

$$\frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$= \frac{0\,(x+6) - 1\cdot 8}{(x+6)^2}$$

$$= \frac{-8}{(x+6)^2}$$

➢A

Velocity and Acceleration

Let the speed of an object given by the equation $s = f(t)$ then the velocity $v = f'(t)$ and acceleration is given by $a = f''(t)$

11. If the speed of an object at any time is given by $s(t) = 16t - 5t^2 + 4$, then find the equation of its instantaneous velocity.

A $16 - 10t$ **B** $16t - 5$

C $16 - 5t$ **D** $16 - t$

$s(t) = 16t - 5t^2 + 4$

$v = s'(t) = 16 - 10t$

➢A

12. If the displacement of an object at any time is given by $s(t) = 24t + t^2$, find the instantaneous velocity at t=3

A 81 **B** 26

C 30 **D** 6

$$s(t) = 24t + t^2$$
$$v = s'(t) = 24 + 2t$$
$$s'(3) = 24 + 2(3)$$
$$= 30$$

➤ C

13. Find the acceleration of an object if its speed at any time is given by $s(t) = 2x^3 + 4 + x^2$

A $6x^2 + 2x$ **B** $6x^2 + 2x + 4$

C $12x^2 + 2x$ **D** $12x + 2$

$$s(t) = 2x^3 + x^2 + 4$$
$$v \to s'(t) = 6x^2 + 2x$$
$$a \to s''(t) = 12x + 2$$

➤ D

14. If $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$ then find $\frac{d}{dx}(\sin x \cos x)$

A $\sin^2 x - \cos^2 x$ **B** $\sin 2x$

C $\cos^2 x + \sin^2 + x$ **D** $\cos 2x$

$$f(x) = \sin x \text{ and } g(x) = \cos x$$
$$\frac{d}{dx}[f(x)\, g(x)] = f'(x)\, g(x) + f(x)\, g'(x)$$
$$f = \sin x \qquad f' = \cos x$$
$$g = \cos x \qquad g' = -\sin x$$
$$\cos x \cos x + (-\sin x)(\sin x)$$
$$\cos^2 x - \sin^2 x$$

➤ D

15. Find the acceleration of an object after 2 seconds if its displacement at any time is given by $s(t) = x^2(x^3 + 1)$

A 158 **B** 36

C 84 **D** 162

$$s(t) = x^2(x^3 + 1) = x^5 + x^2$$
$$v \to s'(t) = 5x^4 + 2x$$
$$a \to s''(t) = 20x^3 + 2$$
$$s''(2) = 20(2)^3 + 2$$
$$= 160 + 2$$
$$= 162$$

➤ D

Extreme Values of Functions

- Extreme values are any maximum or minimum values on a given interval and where these values are located .
- Critical point is a point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist.
- The extreme values found at the end points and at critical points.

16. The critical point of the function $f(x) = 4x - 3x^2 + 5$ is at $x = ...$

A $\frac{3}{2}$ **B** $\frac{-2}{3}$

C $\frac{2}{3}$ **D** $\frac{-3}{2}$

$$f(x) = 4x - 3x^2 + 5$$
$$f'(x) = 4 - 6x$$

To find the critical point let $f'(x) = 0$

$$4 - 6x = 0$$
$$6x = 4$$
$$x = \frac{4}{6}$$
$$x = \frac{2}{3}$$

➤ C

Example:

Find the maximum value of the function $f(x)$ in the interval $[0, 3]$ where $f(x) = \frac{x^3}{3} - 3x^2 + 8x - 4$

Step 1: Find the critical points

$f(x) = \frac{x^3}{3} - 3x^2 + 8x - 4$

$f'(x) = x^2 - 6x + 8$

let $f(x) = 0$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\rightarrow x = 4 \text{ or } x = 2$

Step 2: Compare x values to the interval

$x = 4 \notin [0, 3] \rightarrow$ rejected

$x = 2 \in [0, 3] \rightarrow$ accepted

Step 3: Find the values of the endpoints and the critical points: $0, 2, 3$

$f(0) = \frac{0^3}{3} - 3(0)^2 + 8(0) - 4$

$= -4$

$f(2) = \frac{2^3}{3} - 3(2)^2 + 8(2) - 4$

$= \frac{8}{3}$

$f(3) = \frac{3^3}{2} - 3(3)^2 + 8(3) - 4$

$= 6.5$

The maximum value of the function $f(x)$ in the interval $[0, 3]$ is 6.5

17. The maximum value of the function $f(x) = 3x^3 - 16x$ in the internal $[-5, 0]$ is at

A $x = \pm\frac{4}{3}$

B $x = \frac{-4}{3}$

C $x = \frac{4}{3}$

D $x = -\frac{3}{4}$

Step 1: Find the critical point

$f(x) = 3x^3 - 16x$

$f'(x) = 9x^2 - 16$

let $f'(x) = 0$

$9x^2 - 16 = 0$

$(3x-4)(3x+4) = 0$

$x = \pm\frac{4}{3}$ ➤ A

Step 2: Compare x values to the interval

$x = \frac{4}{3} \notin [-5, 0]$ rejected

$x = \frac{-4}{3} \in [-5, 0]$ accepted

➤ **B**