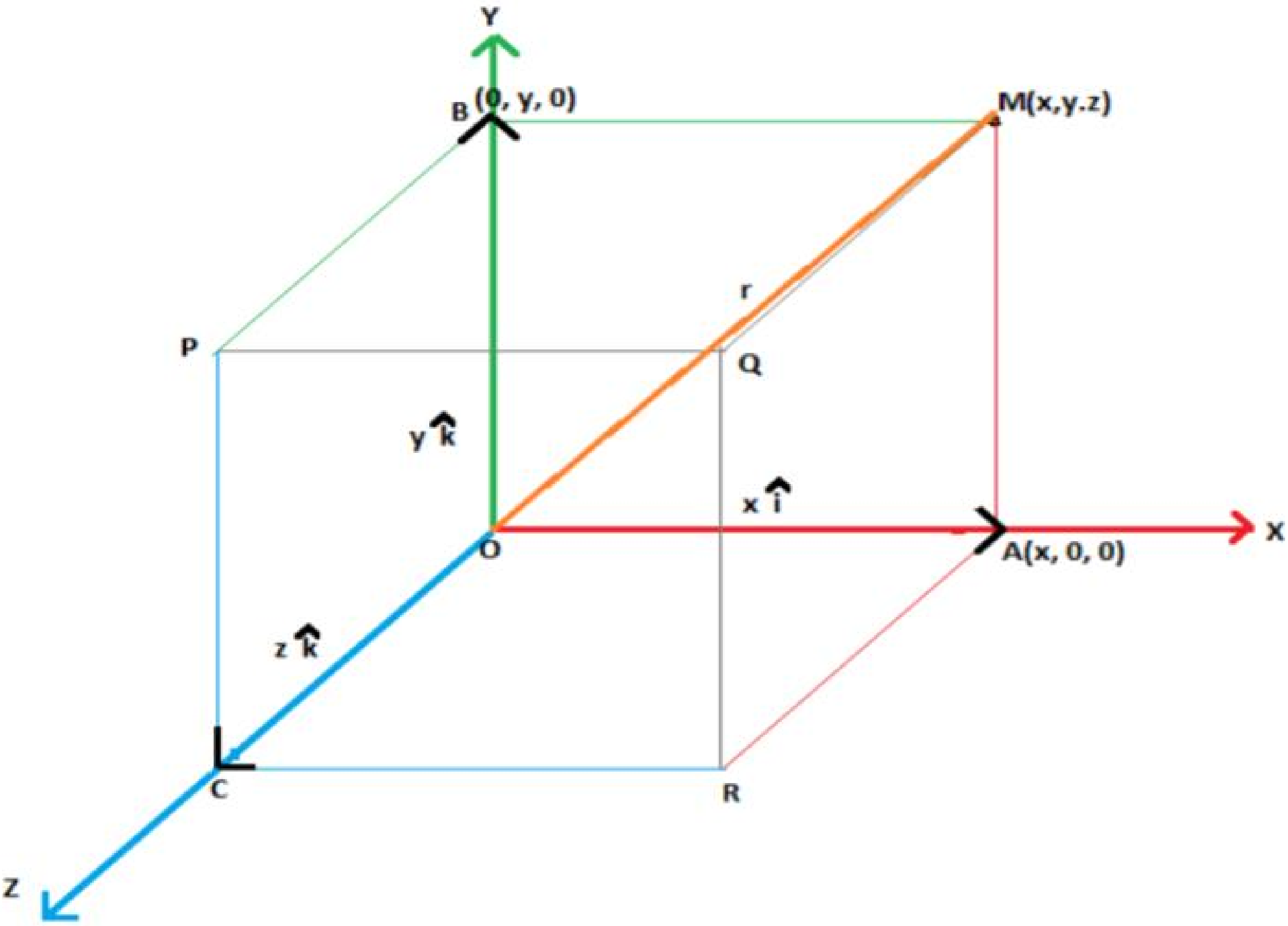
CHAPTER (16)
VECTORS



- **Physical quantities** (or scalar quantities) speed, mass, time can be completely described by a single number called scalar. This number indicates the magnitude or size of the quantity.
- **Vector** is a quantity that has both magnitude and direction. The velocity of a car is a vector that describes both the speed and direction of the car.

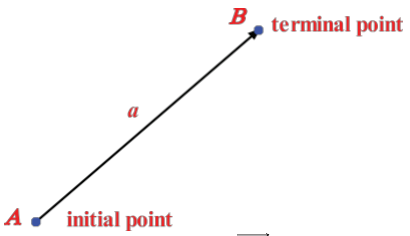
Example:

Determine the vector quantity from the following quantities:

A Time **B** Distance **C** Force **D** Mass
E Speed **F** Displacement **G** Energy **H** Gravity
i Pressure **J** Velocity **K** Area **L** Temperature
M Momentum

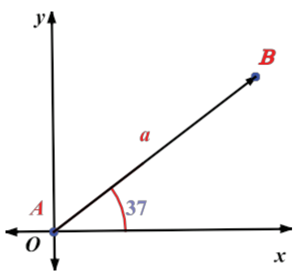
Scalar Quantities magnitude only	Vector Quantities magnitude and direction
Time Distance Mass Speed Energy Area Temperature	Force Displacement Gravity Pressure Velocity Momentum

- Consider the directed line segment with an initial point A (also known as the tail) and terminal point B (also known as head) shown.



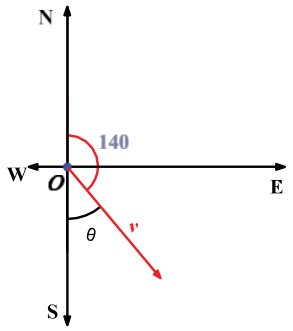
This vector is denoted by $\overrightarrow{AB}$ $\vec{a}$ or a

- The direction of a vector is the directed angle between the vector and the positive x-axis.



- The direction of a vector can also be given as bearing

A quadrant bearing angle is a directional measurement between $0°$ and $90°$ east or west of the north-south line

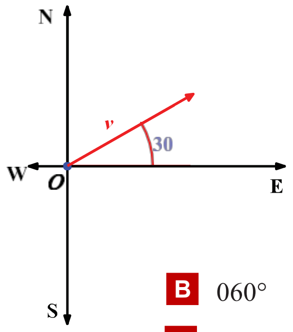


v is $40°$ east of south (or south-east) written as $S40°E$

A true bearing: is a directional measurement where the angle is measured clockwise from North.

True bearings are always given using three digits $25°$ written as $025°$

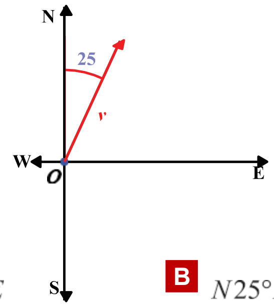
16.1 Find the true bearing of the vector v



A $030°$ **B** $060°$
C $090°$ **D** $045°$

$90 - 30 = 60 \rightarrow 060°$ **B**

16.2 Find the quadrant bearing of the vector v



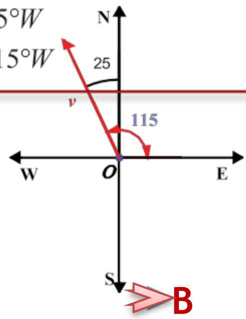
A $N65°E$ **B** $N25°E$
C $N25°W$ **D** $W65°S$

The angle is in quadrant I, so the angle is from north to east $\rightarrow N25°E \rightarrow$ **B**

16.3 Find the quadrant bearing if the measure of the angle of the vector v in the standard position is $115°$

A $N25°E$ **B** $N25°W$
C $N115°E$ **D** $N115°W$

Draw the vector



$115 - 90 = 25°$

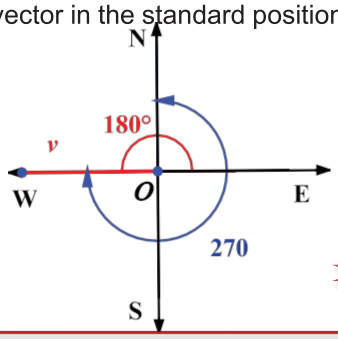
from north to the west **B**

16.4 Find the true bearing of the vector v if its angle in the standard position is $180°$

A $090°$ **B** $180°$

C $270°$ **D** $360°$

The true bearing is measured clockwise from north graph the vector in the standard position $180°$

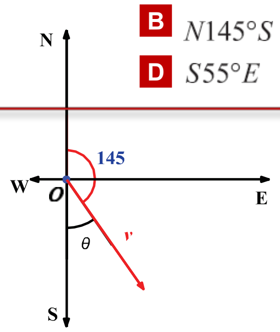


C

16.5 Find the quadrant bearing of a vector if the true bearing is $145°$

A $S35°E$ **B** $N145°S$

C $E55°S$ **D** $S55°E$



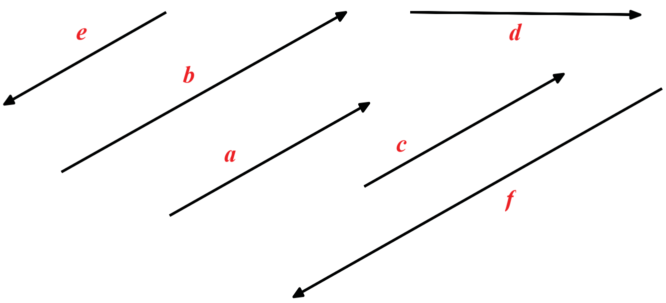
The true bearing is $145°$ from north, so the angle to the south axis is $180-145=35°$

Since the angle is in quadrant IV, therefore the quadrant bearing is from south to east $S35°E$

A

Vector Types

In your operations with vectors, you will need to be familiar with following vector types.

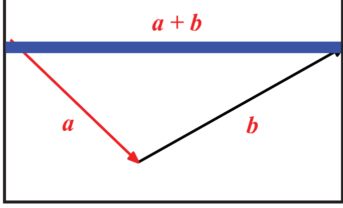
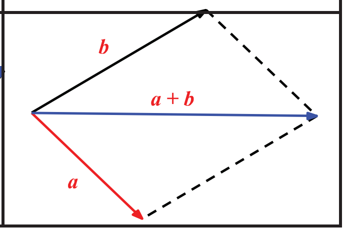


- **Parallel vectors** have the same or opposite direction but not necessarily the same magnitude
  $a \parallel b \parallel c \parallel e \parallel f$
- **Equivalent vectors** have the same magnitude and direction $a=c$

  $a \neq b \quad why?$

  $a \neq d \quad why?$

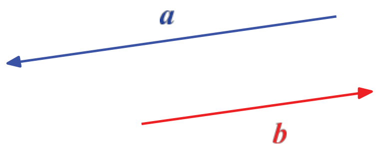
- **Opposite vectors** have the same magnitude but opposite direction $a=-e$
- **Resultant vector,** when two or more vectors are added, then sum is a single vector called the resultant.

Finding Resultants	
Triangle Method Head to Tail	Parallelogram Method Tail to Tail
	

- Two or more vectors with a sum that is a vector r are called **components** for r. While components can have any direction, it is often useful to express or resolve a vector into two perpendicular components. The rectangular components are horizontal (x) and vertical (y).

$$|x| = N\cos\theta$$
$$|y| = N\sin\theta$$

16.6 Describe the two vectors



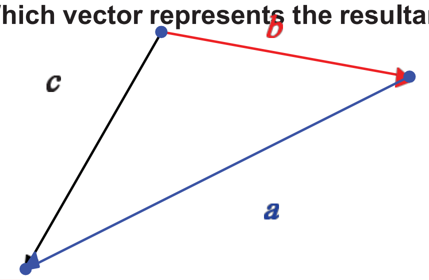
A Congruent **B** Equivalent

C Opposite **D** parallel

The two vectors are parallel, opposite in direction but doesn't equivalent $\rightarrow$ parallel

D

16.7 Which vector represents the resultant?



A a **B** b

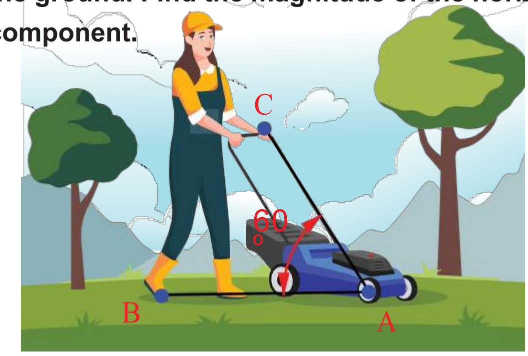
C c **D** a, b, c

In the triangle method head to tail is used $b \rightarrow a$,

then the resultant is c

C

16.8 **Yasmine is pushing the handle of a lawn mower with a force of 400 newtons at an angle of 60° with the ground. Find the magnitude of the horizontal component.**



A 200 B 400

C $200\sqrt{3}$ D $400\sqrt{3}$

Horizontal component $= |x| = N\cos\theta$

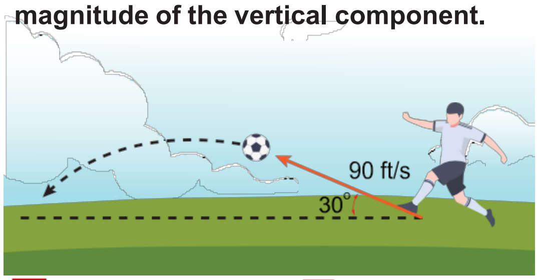
$= 400\,\mathrm{Cos}\,60$

$= 400 \times \dfrac{1}{2}$

$= 200$

A

16.9 **For a field goal attempt, a football is kicked with velocity show in the diagram below. Find the magnitude of the vertical component.**



A $90\sqrt{3}$ B 90

C 45 D $45\sqrt{3}$

Vertical component $|y| = N\,\mathrm{Sin}\,\theta$

$= 90\,\mathrm{Sin}\,30$

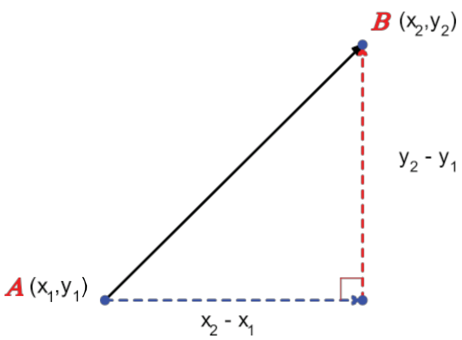
$= 90 \times \dfrac{1}{2}$

$= 45$

C

Vectors in the Coordinate Plane

Components form a vector $\overrightarrow{AB}$ with initial point $A(x_1\ y_1)$ and terminal point $B(x_2\ y_2)$ is given by $\langle x_2 - x_1,\ y_2 - y_1\rangle$



- Magnitude of a vector in the coordinate plane is given by $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- If v has a component from $\langle a,\ b\rangle$ then $|v| = \sqrt{a^2 + b^2}$

16.10 **Find the magnitude of $\overrightarrow{AB}$ with initial point $A(7,\ 9)$ and terminal point $B(4,\ 5)$**

A 25 B 5

C $\sqrt{3}$ D 10

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(7-4)^2 + (9-5)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$
$$= 5$$

B

16.11 **Find the component form of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$**

A $\langle 7,\ -7\rangle$ B $\langle -7,\ 7\rangle$

C $\langle 6,\ -8\rangle$ D $\langle -8,\ 6\rangle$

$$\overrightarrow{AB} = \langle x_2 - x_1,\ y_2 - y_1\rangle$$
$$= \langle 3 - (-4),\ -5 - 2\rangle$$
$$= \langle 7,\ -7\rangle$$

A

16.12 **Which vector has magnitude equal 9**

A $\langle 3\sqrt{3},\ 3\rangle$ B $\langle 3\sqrt{3},\ 3\sqrt{6}\rangle$

C $\langle \sqrt{5},\ 4\rangle$ D $\langle 4,\ 2\rangle$

$$|v| = \sqrt{a^2 + b^2}$$
$$|v| = \sqrt{\left(3\sqrt{3}\right)^2 + \left(3\sqrt{6}\right)^2}$$
$$= \sqrt{9\cdot 3 + 9\cdot 6}$$
$$= \sqrt{27 + 54}$$
$$= \sqrt{81}$$
$$= 9$$

B

Vector Operations

$if\ \ a = \langle a_1, a_2\rangle\ and\ b = \langle b_1, b_2\rangle$

		Example K=6 a=<2‹4> b= <3‹7>
Addition $a+b$	$\langle a_1+b_1, a_2+b_2\rangle$	$\langle 2+3, 4+7\rangle = \langle 5, 11\rangle$
Subtraction $a-b$	$\langle a_1-b_1, a_2-b_2\rangle$	$\langle 2-3, 4-7\rangle = \langle -1, -3\rangle$
Scalar multipliction ka	$\langle ka_1, ka_1\rangle$	$6a = \langle 6\cdot 2, 6\cdot 4\rangle$ $= \langle 12, 24\rangle$

16.13 If $-3v = \langle 12, -15\rangle$, **then find** v

A $\langle 4, -5\rangle$ **B** $\langle -36, 45\rangle$
C $\langle -4, 5\rangle$ **D** $\langle 36, -45\rangle$

$$-3v = \langle 12, -15\rangle$$
$$\frac{-3}{-3}v = \left\langle \frac{12}{-3}, \frac{-15}{-3}\right\rangle$$
$$= \langle -4, 5\rangle$$

➢C

16.14 If $A = \langle 1, 3\rangle, B = \langle 2, 4\rangle$ **then find** $2B - A$

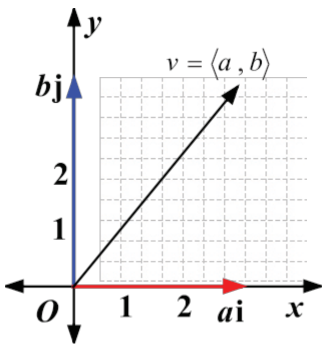
A $\langle 5, 3\rangle$ **B** $\langle 3, 5\rangle$
C $\langle -2, -2\rangle$ **D** $\langle 2, 2\rangle$

$$2B - A$$
$$2\langle 2, 4\rangle - \langle 1, 3\rangle$$
$$\langle 4, 8\rangle - \langle 1, 3\rangle = \langle 3, 5\rangle$$

➢B

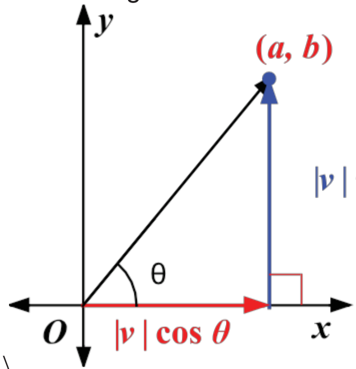
Unit Vectors

- A vector that has a magnitude of 1 unit is called unit vector
$$u = \frac{v}{|v|}$$
- The unit vectors in the direction of the positive ***x*-axis** and ***y*-axis** are denoted by $i = \langle 1, 0\rangle$ and $j = \langle 0, 1\rangle$ respectively. Vectors ***i*** and ***j*** are called standard unit vectors.



The vector sum $ai + bj$ is called linear combination.

- The vector v can be written in component form or as a linear combination of i and j using the magnitude and direction angle of the vector.



$$v = \langle a, b\rangle$$
$$= \langle |v|\cos\theta, |v|\sin\theta\rangle$$
$$= |v|(\cos\theta)i + |v|(\sin\theta)j$$

- It also follows that the direction angle θ of vector $v = \langle a, b\rangle$ an be found by solving trigonometric equation
$$\tan\theta = \frac{|v|\mathrm{Sin}\,\theta}{|v|\mathrm{Cos}\,\theta}\ \ or\ \ \tan\theta = \frac{b}{a}$$
- On the other hand for a given $v = \langle a, b\rangle$, then θ is given by

$\theta = \tan^{-1}\frac{b}{a}$	***a*** is positive
$\theta = \tan^{-1}\frac{b}{a} + \pi$	***a*** is negative

16.15 Find a unit vector u **with the same direction as** $v = \langle 5, 12\rangle$

A $\left\langle \frac{12}{13}, \frac{5}{13}\right\rangle$ **B** $\left\langle \frac{13}{12}, \frac{13}{5}\right\rangle$
C $\left\langle \frac{5}{13}, \frac{12}{13}\right\rangle$ **D** $\left\langle \frac{13}{5}, \frac{13}{12}\right\rangle$

$$|v| = \sqrt{5^2 + 12^2}$$
$$= \sqrt{169}$$
$$= 13$$
$$u = \frac{v}{|v|}$$
$$= \frac{\langle 5, 12\rangle}{13}$$
$$= \left\langle \frac{5}{13}, \frac{12}{13}\right\rangle$$

➢C

16.16 Write the component form of the vector
$v = \langle 3i - 5j \rangle$

A $\langle -3, 5 \rangle$
B $\langle 3, -5 \rangle$
C $\langle 3, 5 \rangle$
D $\langle 5, -3 \rangle$

$$v = ai + bj \rightarrow v = \langle a, b \rangle$$
$$3i - 5j \rightarrow \langle 3, -5 \rangle$$

➢B

16.17 Let $\overline{DE}$ be the vector with initial point $D(-2, 3)$ and terminal point $E(4, 5)$. Write $\overline{DE}$ as a linear combination of the vectors *i* and *j*

A $2i + 6j$
B $-2i + 3j$
C $4i + 5j$
D $6i + 2j$

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle 4-(-2), 5-3 \rangle$$
$$= \langle 6, 2 \rangle$$
$$= 6i + 2j$$

➢D

16.18 Find the component form of the vector v with magnitude 10 and $\theta = 120°$

A $\langle 5, -5\sqrt{3} \rangle$
B $\langle -5, 5\sqrt{3} \rangle$
C $\langle 5, 5\sqrt{3} \rangle$
D $\langle 5\sqrt{3}, -5$

By graphing the vector, we find that x or a is negative and y or b is positive therefore only option has negative x and positive y.

By solving

$$v = \langle |v|\cos\theta, |v|\sin\theta \rangle$$
$$= \langle 10\cos 120, 10\sin 120 \rangle$$
$$= \left\langle 10\left(\frac{-1}{2}\right), 10\frac{\sqrt{3}}{2} \right\rangle$$
$$= \langle -5, 5\sqrt{3} \rangle$$

➢B

16.19 Write the vector $v = \langle 5, 8 \rangle$ as a linear combination of the vectors *i* and *j*

A $5j + 8i$
B $5j - 8i$
C $5i + 8j$
D $5i - 8j$

$$v = \langle a, b \rangle \rightarrow ai + bi$$
$$v = \langle 5, 8 \rangle \rightarrow 5i + 8j$$

➢C

- **Dot Product**

Dot product of $a = \langle a_1, a_2 \rangle$ and $b = \langle b_1, b_2 \rangle$ is defined as $a \cdot b = a_1 b_1 + a_2 b_2$

- **Orthogonal Vectors (Perpendicular)**

The vectors a and b are orthogonal if and only if $a \cdot b = 0$

- **Angle between two Vectors**

If θ is the angle between nonzero vectors ***a*** and ***b***, then $\cos\theta = \frac{a \cdot b}{|a|\,|b|}$

16.20 Find the dot product of u and v, if
$u = \langle 2, 5 \rangle$ $v = \langle 8, 4 \rangle$

A 2
B 4
C 32
D 36

$$a \cdot b = a_1 b_1 + a_2 b_2$$
$$u \cdot v = 2\times 8 + 5\times 4$$
$$= 16 + 20$$
$$= 36$$

➢D

16.21 Determine the perpendicular vectors
$t = \langle 3, 6 \rangle, u = \langle -2, 7 \rangle, v = \langle -4, 2 \rangle, w = \langle 4, 6 \rangle$

A $t \cdot v$
B $u \cdot v$
C $w \cdot v$
D $w \cdot t$

Vectors are perpendicular if and only if $a \cdot b = 0$

$$t \cdot v = 3(-4) + 6(2)$$
$$= -12 + 12$$
$$= 0$$

➢A

16.22 Find the angle θ between the vectors

$u = \langle -2, 0\rangle$ *and* $v\langle -3, -3\rangle$

A $45°$ B $60°$

C $30°$ D $120°$

$$u \cdot v = (-2)(-3) + (0)(-3)$$
$$= 6 + 0$$
$$= 6$$

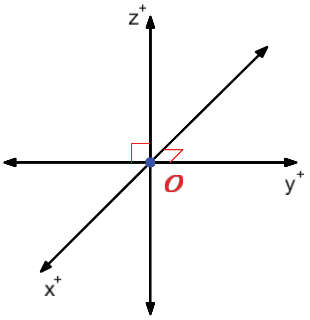
$$|u||v| = \sqrt{(-2)^2 + 0^2} \times \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{4} \times \sqrt{18} = \sqrt{72}$$
$$= 6\sqrt{2}$$

$$\text{Cos}\,\theta = \frac{u \cdot v}{|u||v|} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow \theta = 45°$$

A

Unit Vectors

In the three-dimensional coordinate system, a third axis called z-axis that passes through the origin and is perpendicular to both the x-axis and y-axis



Distance and Midpoint Formula in Space

- The distance between points $A(x_1, y_1, z_1)$ *and* $B(x_2, y_2, z_2)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- The midpoint M of $\overline{AB}$ is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

16.23 Find the length of the segment between the endpoints $A(-4, 10, 4)$, $B(1, 0, 9)$

A $6\sqrt{5}$ B $5\sqrt{6}$

C 10 D 5

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(1 - (-4))^2 + (0 - 10)^2 + (9 - 4)^2}$$
$$= \sqrt{5^2 + (-10)^2 + 5^2}$$
$$= \sqrt{25 + 100 + 25}$$
$$= \sqrt{150}$$
$$= \sqrt{6 \cdot 25}$$
$$= 5\sqrt{6}$$

B

16.24 If $(3, 4, 4)$ is the midpoint of AB, $A(-3, 2, 8)$ *and* $B(9, 6, k)$, **then find the value of k**

A 4 B 6

C 2 D 0

$$\frac{z_1 + z_2}{2} = M$$
$$\frac{8 + k}{2} = 4$$
$$8 + k = 8$$
$$k = 0$$

D

16.25 Classify the triangle ABC with vertices $A(3, 6, 1)$, $B(5, 7, 4)$ *and* $C(1, 5, -2)$

A Right B Isosceles

C Equilateral D Neither

$$AB = \sqrt{(5-3)^2 + (7-6)^2 + (4-1)^2} \quad = \sqrt{2^2 + 1^2 + 3^2} \quad = \sqrt{14}$$
$$AC = \sqrt{(3-1)^2 + (6-5)^2\,(1-(-2))^2} \quad = \sqrt{2^2 + 1^2 + 3^2} \quad = \sqrt{14}$$
$$BC = \sqrt{(5-1)^2 + (7-5)^2 + (4-(-2))^2} \quad = \sqrt{4^2 + 2^2 + 6^2} \quad = \sqrt{56}$$

$AB = AC$ then it is an isosceles triangle

B

Express Vectors in Space

To find the component form of the directed line segment from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ is

$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle$

The unit vectors are: $i = \langle 1, 0, 0\rangle$, $j = \langle 0, 1, 0\rangle$, $k = \langle 0, 0, 1\rangle$

The linear combination of $v = \langle v_1, v_2, v_3\rangle$ is $v = v_1\, i + v_2\, j + v_3\, k$

The magnitude of the vector $v = \langle v_1, v_2, v_3 \rangle$

$$|v| = \sqrt{v_1^{\,2} + v_2^{\,2} + v_3^{\,2}}$$

The unit vector $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$ is $u = \dfrac{v}{|v|}$

16.26 Find the component form of $\overrightarrow{AB}$ if

$A(-4, -2, 1)$ *and* $B(3, 6, -6)$

A $\langle -7, -8, 7 \rangle$ **B** $\langle 7, 8, -7 \rangle$

C $\langle -7, 8, -7 \rangle$ **D** $\langle 7, 8, 7 \rangle$

$$\overrightarrow{AB} = \langle 3-(-4), 6-(-2), -6-1 \rangle$$
$$= \langle 7, 8, -7 \rangle$$

B

16.27 The magnitude of the vector $v = 7i + 8j - 7k$ is

A $\sqrt{22}$ **B** 162

C 22 **D** $9\sqrt{2}$

$$|v| = \sqrt{7^2 + 8^2 + (-7)^2}$$
$$= \sqrt{49 + 64 + 49}$$
$$= \sqrt{162}$$
$$= \sqrt{2 \cdot 81}$$
$$= 9\sqrt{2}$$

D

16.28 Find the unit vector in the direction of the vector $v = \langle 3, 2, \sqrt{3} \rangle$

A $\left\langle \dfrac{3}{4}, \dfrac{1}{2}, \dfrac{\sqrt{3}}{4} \right\rangle$ **B** $\left\langle \dfrac{4}{3}, 2, \dfrac{4}{\sqrt{3}} \right\rangle$

C $\left\langle \dfrac{3}{\sqrt{8}}, \dfrac{2}{\sqrt{8}}, \dfrac{\sqrt{3}}{\sqrt{8}} \right\rangle$ **D** $\left\langle \dfrac{\sqrt{8}}{3}, \dfrac{\sqrt{8}}{2}, \dfrac{\sqrt{8}}{\sqrt{3}} \right\rangle$

$$|v| = \sqrt{3^3 + 2^2 + \left(\sqrt{3}\right)^2}$$
$$= \sqrt{9 + 4 + 3}$$
$$= \sqrt{16}$$
$$= 4$$

$$u = \frac{v}{|v|} = \frac{3, 2, \sqrt{3}}{4}$$
$$= \left\langle \frac{3}{4}, \frac{1}{2}, \frac{\sqrt{3}}{4} \right\rangle$$

A

Vector Operations in Space

If $a = \langle a_1, a_2, a_3 \rangle$, $b = \langle b_1, b_2, b_3 \rangle$

Vector operation	Result	Example K = 5 a = <2,1,0> b= <-1,2,7>
Addition $\boldsymbol{a + b}$	$\langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$	$\langle 2 + (-1), 1 + 2, 0 + 7 \rangle$ $\langle 1, 3, 7 \rangle$
Subtraction $\boldsymbol{a - b}$	$\langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$	$\langle 2 - (-1), 1 - 2, 0 - 7 \rangle$ $\langle 3, -1, -7 \rangle$
Scalar multiplication $\boldsymbol{ka}$	$\langle k a_1, k a_2, k a_3 \rangle$	$\langle 5(2), 5(1), 5(0) \rangle$ $\langle 10, 5, 0 \rangle$

Dot Products of Vectors in Space

The dot product of $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$ is defined as $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular if and only if $a \cdot b = 0$

16.29 Let $y = \langle -1, 4, -4 \rangle$ and $z = \langle -2, 0, 5 \rangle$, then find

$4y + 2z$

A $6\langle -3, 4, 1 \rangle$ **B** $8\langle 1, -4, 9 \rangle$

C $\langle -8, 16, -6 \rangle$ **D** $\langle 8, -16, 6 \rangle$

$$4y + 2z = 4\langle -1, 4, -4 \rangle + 2\langle -2, 0, 5 \rangle$$
$$= \langle -4, 16, -16 \rangle + \langle -4, 0, 10 \rangle$$
$$= \langle -8, 16, -6 \rangle$$

C

16.30 Let $y = \langle 3, -6, 2 \rangle$ $w = \langle -1, 4, -4 \rangle$ $z = \langle -2, 0, 5 \rangle$ then find $2w - z + 3y$

A $\langle -7, -10, 9 \rangle$ **B** $\langle 9, -10, -7 \rangle$

C $\langle 7, 10, 9 \rangle$ **D** $\langle 9, 10, 7 \rangle$

$$2w - z + 3y = 2\langle -1, 4, -4 \rangle - \langle -2, 0, 5 \rangle + 3\langle 3, -6, 2 \rangle$$
$$= \langle -2, 8, -8 \rangle - \langle -2, 0, 5 \rangle + \langle 9, -18, 6 \rangle$$
$$= \langle 9, -10, -7 \rangle$$

B

16.31 Find the dot product of u and v if

$u = \langle -7, 3, -3 \rangle,\ v = \langle 5, 17, 5 \rangle$

A 1 **B** 31

C -1 **D** -1

$$u \cdot v = -7(5) + 3(17) + (-3)(5)$$
$$= -35 + 51 - 15$$
$$= 1$$

A

16.32 If the vectors u and v are orthogonal then find k

$u = \langle 3, -3, 3 \rangle,\ v = \langle 4, k, 3 \rangle$

A -7 **B** 7

C 8 **D** -8

For perpendicular vectors $a \cdot b = 0$

$$u \cdot v = 3 \cdot 4 + (-3) \cdot k + 3 \cdot 3 = 0$$
$$= 12 - 3k + 9 = 0$$
$$= -3k = -21$$
$$k = 7$$

B

16.33 Find the angle θ between u and v

$u = \langle \sqrt{2}, 2, 0 \rangle,\ v = \langle \sqrt{3}, 0, 1 \rangle$

A $120°$ **B** $30°$

C $45°$ **D** $60°$

$$\cos\theta = \frac{u \cdot v}{|u|\,|v|}$$

$$u \cdot v = \sqrt{2} \times \sqrt{3} + (2 \times 0) + (0 \times 1)$$
$$= \sqrt{6} + 0 + 0 = \sqrt{6}$$

$$|u| = \sqrt{(\sqrt{2})^2 + 2^2 + 0^2}$$
$$= \sqrt{2 + 4 + 0}$$
$$= \sqrt{6}$$

$$|v| = \sqrt{(\sqrt{3})^2 + 0^2 + 1^2}$$
$$= \sqrt{3 + 0 + 1}$$
$$= 2$$

$$\cos\theta = \frac{\cancel{\sqrt{6}}}{2 \cdot \cancel{\sqrt{6}}} = \frac{1}{2}$$

$$\cos^{-1}\frac{1}{2} \quad \theta = 60^0$$

D

Cross Product of Vectors in Space

If $a = ai + a_2 j + a_3 k$ and $b = b_1 k + b_2 j + b_3 k$, the cross product of a and b is the vector

$$a \times b = (a_2 b_3 - a_3 b_2) i - (a_1 b_3 - a_3 b_1) j + (a_1 b_2 - a_2 b_1) k$$

To find the product we can apply the formula for calculating the determinant of a 3×3 matrix.

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 & a_3 \\ b_2 & b_3 \end{pmatrix} i - \begin{pmatrix} a_1 & a_3 \\ b_1 & b_3 \end{pmatrix} j + \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \mathbf{k}$$

If $a \times b = c$ the c is perpendicular to a and b

If a and b are adjacent sides in a parallelogram, then $|a \times b|$ equals the area of that parallelogram.

Example:

Find the area of the parallelogram if $a = -4i + j + 8k$ and $b = 3i - 4j - 3k$ are two adjacent sides of the parallelogram.

Solution:

Step 1:

$$: b = \begin{vmatrix} i & j & k \\ -4 & 1 & 8 \\ 3 & -4 & -3 \end{vmatrix}$$
$$= [-3 - (-32)] i - [12 - 24] j + [16 - 3] k$$
$$= 29i + 12j + 13k$$

Step 2:

$$Area = |a \times b| = \sqrt{(29)^2 + 12^2 + 13^2}$$
$$= \sqrt{841 + 144 + 169}$$
$$= \sqrt{1154}$$

16.34 Find $\begin{vmatrix} i & j & k \\ 3 & -2 & 1 \\ -3 & 3 & 1 \end{vmatrix}$

A $-5i + 6j + 3k$ **B** $-5i - 6j + 3k$

C $-i + 6j + 14k$ **D** $-i - 6j + 14k$

This question is the same as:

Find the cross product of

$u = \langle 3, -2, 1 \rangle$ *and* $v = \langle -3, 3, 1 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & -2 & 1 \\ -3 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & 1 \end{pmatrix} i - \begin{pmatrix} 3 & 1 \\ -3 & 1 \end{pmatrix} j + 3\begin{pmatrix} 3 & -2 \\ -3 & 3 \end{pmatrix} k$$
$$= (-2 - 3) i - [3 - (-3)] j + (9 - 6) k$$
$$= -5i - 6j + 3k$$

B

16.35 **Determine the perpendicular vector to both vectors** $u=-2i-j-3k$ **and** $v=5i+j+4k$

A $\langle -1,-7,-3\rangle$ B $\langle 1,7,3\rangle$

C $\langle -1,-7,3\rangle$ D $\langle -1,7,-3\rangle$

Perpendicular vector is $\boldsymbol{u} \times \boldsymbol{v}$

$$u \times v = \begin{pmatrix} i & j & k \\ -2 & -1 & -3 \\ 5 & 1 & 4 \end{pmatrix}$$

$$= \left(-4-(-3)\right)i - \left(-8-(-15)\right)j + \left(-2-(-5)\right)k$$

$$= -i - 7j + 3k$$

$$= \langle -1, -7, 3\rangle$$

C

16.36 **Find the volume of the parallelepiped with adjacent edges** $t = 2i - 2j + 3k,\ u = 3i - 7k$ **and** $v = 2i - 4j$

A 48 B -48

C -64 D 64

$$t\cdot(u\times v) = \begin{pmatrix} 2 & -2 & 3 \\ 3 & 0 & -7 \\ 2 & -4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -7 \\ -4 & 0 \end{pmatrix}(2) - \begin{pmatrix} 3 & -7 \\ 2 & 0 \end{pmatrix}(-2) + \begin{pmatrix} 3 & 0 \\ 2 & -4 \end{pmatrix}(3)$$

$$= [0-28](2) - [0-(-14)](-2) + [-12-0](3)$$

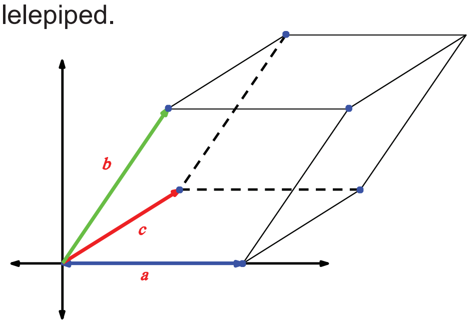
$$= -56 + 28 - 36$$

$$= -64 \rightarrow |-64| = 64$$

D

Triple Scaler Product

Three vectors that lie in different planes but share the same initial point determine the adjacent edges of a **parallelepiped**. The absolute value of the triple scalar product of these vectors represents the volume of the parallelepiped.



If $t = t_1 i + t_2 j + t_3 k$,
$u = u_1 i + u_2 j + u_3 k$ and
$v = v_1 i + v_2 j + v_3 k$, the triple scalar product is given by

$$t\cdot(u\times v) = \begin{pmatrix} t_1 & t_2 & t_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{pmatrix}$$