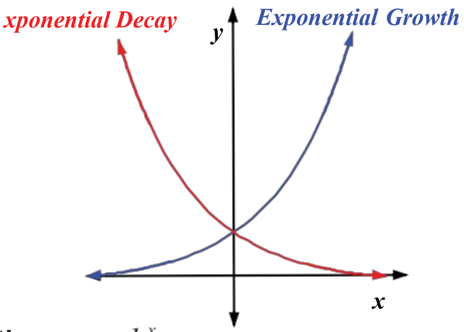
CHAPTER (14)

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Exponential Functions



For the function $y = ab^x$

- If $\boldsymbol{a} > 0$ and $\boldsymbol{b} > 1$ the function represents exponential growth.
- If $\boldsymbol{a} > 0$ and $0 < \boldsymbol{b} < 1$ the function represents exponential decay.
- ***y-intercept*** is $(0, \text{a})$ if $a = 1 \rightarrow y = b^x$ and y-intercept is $(0, 1)$
- Domain is all real numbers $(-\infty, \infty)$
- Range is all positive real numbers $(0, \infty)$
- The horizontal asymptote is $\boldsymbol{y} = 0$

14.1 Find the ***y-intercept*** of the function $y = \left(\frac{1}{3}\right)^x$

A $(1, 0)$ **B** $\left(\frac{1}{3}, 0\right)$

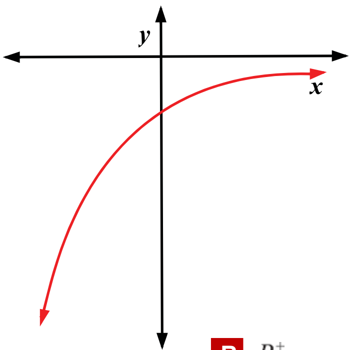
C $(0, 1)$ **D** $\left(0, \frac{1}{3}\right)$

Compare $y = \left(\frac{1}{3}\right)^x$ to $y = ab^x$

$\rightarrow a = 1$

$\rightarrow$ ***y-intercept is*** $(0, a) \rightarrow (0, 1)$ **C**

14.2 Find the domain of the function $f(x)$



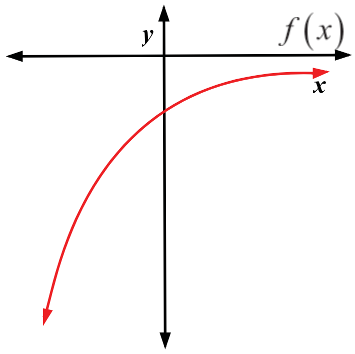
A R **B** R^+

C R^- **D** Z

The domain of an exponential function is all real numbers $\rightarrow R$

You can also use vertical line scan **A**

14.3 Find the range of the function



A R **B** R^+

C R^+ **D** R^-

The range of an exponential function is $\boldsymbol{y} > \boldsymbol{0}$ but ***f(x)*** reflected over the ***x***-axis therefore the range is R^-

You can also use horizontal line scan **C**

14.4 Find the value of x if $7^{4x-2} = 49$

A 2 **B** 1

C 5 **D** 3

$7^{4x-2} = 49$

$7^{4x-2} = 7^2$

$4x - 2 = 2 \qquad x^a = x^b \rightarrow a = b$

$4x = 4$

$x = 1$ **B**

14.5 Find the value of x if $3^{6x-3} = 27^{-3}$

A 1 **B** 2

C -2 **D** -1

$3^{6x-3} = 27^{-3}$

$3^{6x-3} = (3^3)^{-3}$

$3^{6x-3} = 3^{-9}$

$6x - 3 = -9$

$6x = -6$

$x = -1$ **D**

14.6 Find the value of x if $8\left(\frac{2}{3}\right)^{x-11} = 27$

A -2 **B** -4

C 2 **D** 4

$$8\left(\frac{2}{3}\right)^{x+1} = 27$$

$$\left(\frac{2}{3}\right)^{x+1} = \frac{27}{8}$$

$$\left(\frac{2}{3}\right)^{x+1} = \frac{3^3}{2^3}$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{3}{2}\right)^{3}$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{2}{3}\right)^{-3} \qquad \left(\frac{a}{b}\right)^{x} = \left(\frac{b}{a}\right)^{-x}$$

$$x + 1 = -3$$

$$x = -4 \quad \Rightarrow \textbf{B}$$

14.7 Find the value of x if $\frac{3}{-(5)^{x-1}} = -3$

A 1 **B** -1

C 2 **D** -2

We can try the options and substitute for the value of x or solve

$$\frac{3}{-(5)^{x-1}} = -3 \rightarrow \frac{3}{-(5)^{x-1}} = \frac{-3}{1}$$

$$+3(5)^{x-1} = +3$$

$$(5)^{x-1} = +1$$

$$5^{x-1} = (5)^0, \quad 5^0 = 1$$

$$x - 1 = 0$$

$$x = 1 \quad \Rightarrow \textbf{A}$$

Exponential Inequalities

$b > 1$	$b^x > b^y \leftrightarrow x > y$
$0 < b < 1$	$b^x > b^y \leftrightarrow x < y$

14.8 Find the value of x if $2^{x-1} > \frac{1}{32}$

A $x > -4$ **B** $x < -4$

C $x > 6$ **D** $x < 6$

$$2^{x-1} > \frac{1}{32}$$

$$2^{x-1} > \frac{1}{2^5}$$

$$2^{x-1} > 2^{-5}$$

$$x - 1 > -5 \qquad b > 1$$

$$x > -4 \quad \Rightarrow \textbf{A}$$

14.9 Find the value of x if $\left(\frac{1}{2}\right)^{x-1} > \frac{1}{32}$

A $x > -4$ **B** $x < -4$

C $x > -6$ **D** $x < 6$

$$\left(\frac{1}{2}\right)^{x-1} > \frac{1}{32} \rightarrow \left(\frac{1}{2}\right)^{x-1} > \frac{1}{2^5}$$

$$\left(\frac{1}{2}\right)^{x-1} > \left(\frac{1}{2}\right)^{5}$$

$$x - 1 < 5 \qquad 0 < b < 1$$

$$x < 6 \quad \Rightarrow \textbf{D}$$

Logarithmic Functions as Inverse

- A logarithm base b of a positive number x satisfies the following definition.

  For $b > 0,\ b \neq 1,\ \log_b x = y$ if and only if $b^y = x$

  $\log_{10} x = \log x$ if $b = 10$

14.10 Find the value of x if $\log_3 x = 4$

A 12 **B** 81

C 27 **D** 9

$$\log_3 x = 4 \quad \rightarrow \quad 3^4 = x$$

$$= 81 \quad \Rightarrow \textbf{B}$$

14.11 Find the value of x if $\log_x 16 = 4$

A 4 **B** 2

C 3 **D** 12

$$x^4 = 16 \quad \rightarrow \quad x = 2 \quad \Rightarrow \textbf{B}$$

14.12 What is the logarithmic form of the equation $100 = 10^2$

A $\log_{100} 10 = 2$ **B** $\log 2 = 100$

C $\log 100 = 2$ **D** $\log_2 100 = 10$

Compare $x = b^y \rightarrow \log_b x = y$

$100 = 10^2 \rightarrow \log_{10} 100 = 2$

$\rightarrow \log 100 = 2$ ➤ **C**

14.13 What is the exponential equation of $\log_x \frac{8}{27} = 3$

A $\left(\frac{8}{27}\right)^3 = x$ **B** $x^3 = \frac{8}{27}$

C $3^x = \frac{8}{27}$ **D** $3^{\frac{8}{27}} = x$

Compare $\log_b x = y \rightarrow x = b^y$

$\log_x \frac{8}{27} = 3 \rightarrow x^3 = \frac{8}{27}$ ➤ **C**

14.14 What is the logarithmic form of $1 = 3^0$

A $\log_3 1 = 0$ **B** $\log_1 3 = 0$

C $\log_1 0 = 3$ **D** $\log_3 0 = 1$

Compare $x = b^y \rightarrow \log_b x = y$

$1 = 3^0 \rightarrow \log_3 1 = 0$ ➤ **A**

14.15 What is the exponential equation of $\log_x 7 \geq$

A $x^5 \leq 7$ **B** $x^7 \leq 5$

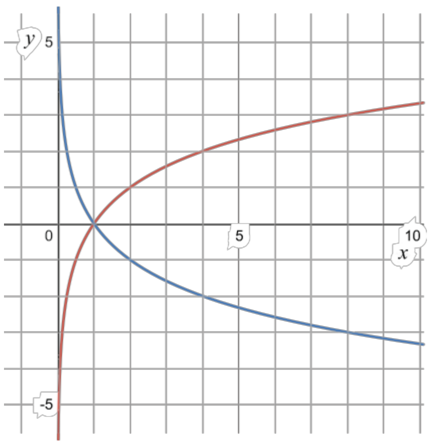
C $5^x \leq 7$ **D** $7^x \leq 5$

Compare $\log_b x \geq y \rightarrow b^y \leq x$

$\log_x 7 \geq 5 \rightarrow x^5 \leq 7$ ➤ **A**

Graphing Logarithmic Function

- The logarithmic function with base b, $\log_b x = y$ is the inverse of $y = b^2$, therefore their graphs are reflections of each other in the line $y = x$



x-intercept $(1, 0)$

vertical asymptote $x = 0$

Domain : $R^+ \rightarrow (0, \infty)$

Range : $R \rightarrow (-\infty, \infty)$

Standard form

$f(x) = \log_b (x - h) + k$

Range : $x - h > 0$

y-intercept $\leftrightarrow f(0)$,

x-intercept $x = h$

14.16 Find the ***x-intercept*** of $f(x) = \log_b x$

A $(0, 1)$ **B** $(1, 0)$

C $(0, -1)$ **D** $(-1, 0)$

This is the parent function of the logarithmic Function and the ***x-intercept*** is $(1, 0)$ ➤ **B**

14.17 Find the y-intercept of $f(x) = \log_2 (x + 4) + 7$

A 9 **B** 0

C 1 **D** 3

The y-intercept is $f(0)$

$f(x) = \log_2 (x + 4) + 7$

$f(0) = \log_2 (0 + 4) + 7$

$= \log_2 (4) + 7$

$= 2 + 7$

$= 9$

➤ **A**

14.18 Find the inverse function of $f(x) = \log_5 (x + 3)$

A $5^x + 3$ **B** $5^x - 3$

C $x^5 + 3$ **D** $x^5 - 3$

$f(x) = \log_5 (x + 3)$

$y = \log_5 (x + 3)$

$5^y = x + 3$

$5^x = y + 3$

$y = 5^x - 3$

$f(x) = 5^x - 3$ ➤ **B**

14.19 Find the range of the function of $f(x) = \log_5(x+3)$

A R B R^+
C R^- D Z

Recall for the logarithmic functions

Domain : $R^+ \to (0,\infty)$

Range : $R \to (-\infty,\infty)$

A

Properties of Logarithms

If $b, x, and\ y > 0, b \neq 1$, and P is a real number then

Product Property	$\log_b xy = \log_b x + \log_b y$
Quotient Property	$\log_b \frac{x}{y} = \log_b x - \log_b y$
Power Property	$\log_b x^p = p \log_b x$

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b b^x = x$

14.20 Evaluate the logarithm $\log_7 7^{35}$

A 49 B 7
C 35 D 5

Use $\log_b b^x = x \to \log_7 7^{35} = 35$

C

14.21 Evaluate the logarithm $\log_9 3^{24}$

A 24 B 8
C 27 D 12

$$\log_9 3^{24} = \log_9 3^{2\times12}$$
$$= \log_9 (3^2)^{12}$$
$$= 12\log_9 9$$
$$= 12 \times 1 = 12$$

D

14.22 Determine the different value

A $\log_3 27$ B $\log_5 125$
C $\log_2 16$ D $\log_4 64$

option C) $\log_2 16 = 4$

Since $2^4 = 16$

other options equals 3

C

14.23 Evaluate $\log_{25} 125$

A $\frac{2}{3}$ B $\frac{3}{2}$
C $\frac{4}{3}$ D $\frac{3}{4}$

$$\log_{25} 125 = \log_{25} 25 \times 5$$
$$= \log_{25} 25 + \log_{25} 5$$
$$= 1 + \frac{1}{2} \qquad \sqrt{25} = 25^{\frac{1}{2}} = 5$$
$$= 1\frac{1}{2}$$
$$= \frac{3}{2}$$

B

14.24 Simplify $3\log_7 x - \log_7(5x+2)$

A $\log_7 \frac{5x+2}{x^3}$ B $\log_7 \frac{x^3}{5x+2}$
C $\log_7 \frac{x}{5x+2}$ D $\log_7 x^3(5x+2)$

$$3\log_7 x - \log_7(5x+2) = \log_7 x^3 - \log_7(5x+2)$$
$$= \log_7 \frac{x^3}{5x+2}$$

B

14.25 Evaluate $\log_5 \frac{1}{125}$

A 3 B -3
C $\frac{1}{3}$ D $\frac{-1}{3}$

$$= \log_5 5^{-3}$$
$$= -3\log_5 5$$
$$= -3 \times 1$$
$$= -3$$

B

14.26 Evaluate $4\log_4 16 + \log_2 \frac{1}{32}$

A 13 **B** -13

C 11 **D** 3

$$4\log_4 16 + \log_2 \frac{1}{32} = 4 \times 2 + \log_2 (2)^{-5}$$
$$= 8 + (-5)\log_2 2$$
$$= 8 - 5 \times 1$$
$$= 3$$

D

14.27 Simplify $(x+5) - \log x^3 + 4\log x$

A $\log x(x+5)$ **B** $\log(x+5)$

C $\log \frac{x}{x+5}$ **D** $\log \frac{x+5}{x}$

$$\log(x+5) - \log x^3 + 4\log x = \left[\log(x+5) - \log x^3\right] + 4\log x$$
$$= \log \frac{x+5}{x^3} + \log x^4$$
$$= \log \frac{(x+5)\,x^4}{x^3}$$
$$= \log x(x+5)$$

A

14.28 Evaluate $\log_{1000} 10$

A $\frac{-1}{3}$ **B** $\frac{1}{3}$

C 3 **D** -3

$$10^3 = 1000 \rightarrow \sqrt[3]{1000} = 10$$
$$\log_{1000} 10 = \log_{1000} 1000^{\frac{1}{3}}$$
$$= \frac{1}{3}\log_{1000} 1000$$
$$= \frac{1}{3} \times 1$$
$$= \frac{1}{3}$$

B

14.29 If $f(x) = \log x,\ 10 \le x \le 100$,
then determine the correct inequality.

A $1 \le f(x) \le 2$ **B** $0 \le f(x) \le 1$

C $1 \le f(x) \le 10$ **D** $10 \le f(x) \le 100$

$$10 \le x \le 100$$
$$f(10) \le f(x) \le f(100)$$
$$\log 10 \le f(x) \le \log 100$$
$$1 \le f(x) \le 2$$

A

14.30 Find the value of x if $-2 + 3\log(x+4) = 16$

A 68 **B** 60

C 28 **D** 36

$$-2 + 3\log_2(x+4) = 16$$
$$3\log_2 x + 4 = 18$$
$$\log_2 x + 4 = 6$$
$$2^6 = x + 4$$
$$64 = x + 4$$
$$x = 60$$

A

14.31 Solve the logarithmic inequality

$\log_8 4x > \log_8 (6x-4)$

A $x < 2$

B $x > 2$

C $0 < x < \frac{2}{3}$

D $\frac{2}{3} < x < 2$

Step **1**, since $b > 1$

$4x > 6x - 4$
$4 > 2x$
$2 > x$

Step **2**, the logarithms should be greater than zero

$\log_8 4x$
$\rightarrow 4x > 0$
$x > 0$
$\log_8 (6x-4)$
$\rightarrow 6x - 4 > 0$
$6x > 4$
$x > \frac{2}{3}$

Option ***D*** satisfies the three inequalities.

D