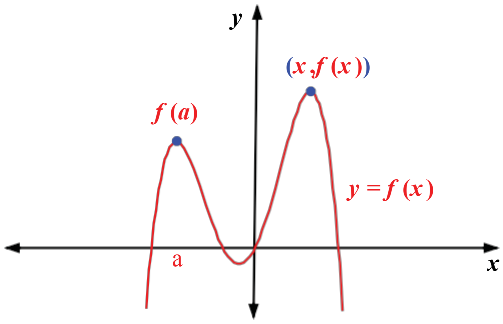
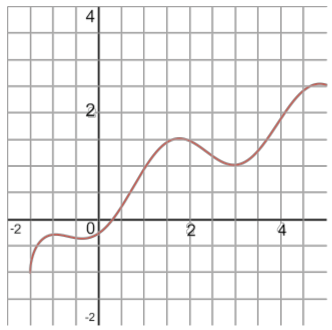
CHAPTER (13)

ANALYSING GRAPHS OF FUNCTIONS AND RELATIONS

- The graph of f is the graph of the equation $y = f(x)$
- The value of the function is the directed distance y of the graph from the point x on the ***x-axis*** as shown.
- The input values ***(x-values)*** correspond to the domain of the relation. The output values ***(y-values)*** corrspond to the range of the relation.
- ***x*-intercept (zeros)** is the point at which the line crosses the ***x-axis***, to find that point we let $f(x) = 0$ and solve for x
- ***y*-intercept** is the point at which the line crosses the ***y-axis***, to find that point we find $f(0)$

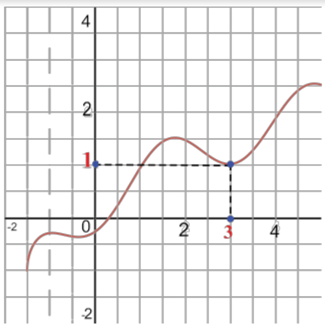


13.1 **Find** $f(3)$



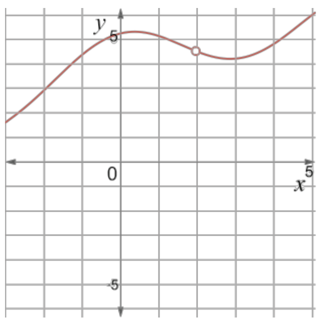
A 1 **B** 3

C 2 **D** 4



A

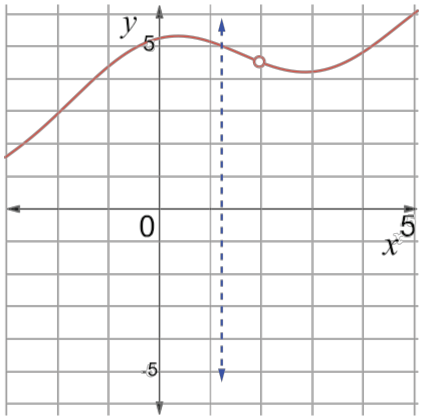
13.2 **Find domain of the function** $f(x)$



A $[1.5, 6] - \{4.5\}$ **B** $[-3, 5] - \{2\}$

C $[-3, 5]$ **D** $[1.5, 6]$

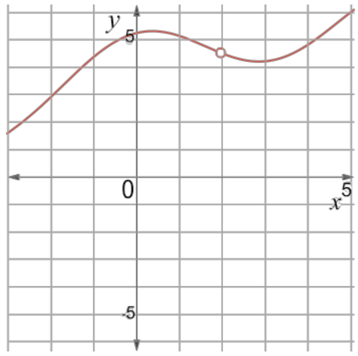
To find the domain we should find the ***x-values.***



By using the vertical line scanning, we find that ***x-values*** started from -3 to 5 but 2 does not belong to the domain

B

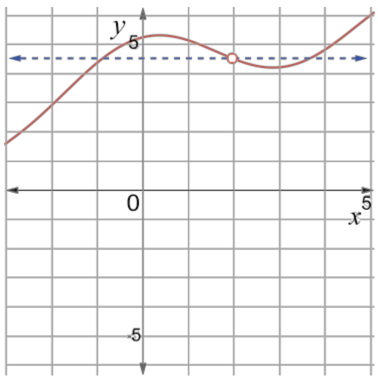
13.3 **Find range of the function** $f(x)$



A $[1.5, 6] - \{4.5\}$ **B** $[-3, 5] - \{2\}$

C $[-3, 5]$ **D** $[1.5, 6]$

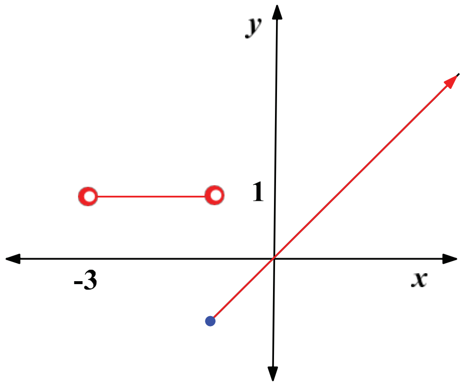
To find the range we should find the ***y-values***



By using the horizontal line scanning, the function starts from 1.5 to 6. Although the point $(2, 4.5)$ is not included but the point $(-1, 4.5)$ is included.

D

13.4 Find the domain of the function $f(x)$



A $[-3, \infty)$ B $[-3, -1] \cup (1, \infty)$

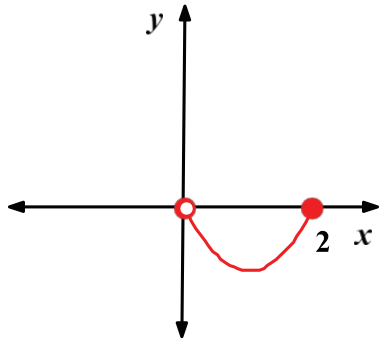
C $(-3, \infty)$ D $(-3, -1) \cup (1, \infty)$

Using the vertical line scanning for x-values, we find that:

- The left side starts from -3 but it is not connected to the graph $\rightarrow (-3$
- -1 is not connected from the left side $\rightarrow -1)$
- The right side starts from -1 and it is connected to the right side $\rightarrow [1$
- The function continue towards $+\infty$ $\rightarrow, \infty)$

$(-3, -1) \cup [1, \infty) = (-3, \infty)$ **C**

13.5 Find the range of the function $f(x)$



A $[-2, 0)$ B $[-2, 0]$

C $(0, 2]$ D $[0, 2]$

Use the horizontal line scanning for ***y-values*** The function starts from -2 to zero. Although the point $(0, 0)$ is not included but the point $(2, 0)$ is included.

B

13.6 Find the y-intercept of $f(x) = 3x^3 + 2x - 4$

A $(0, -4)$ B $(-4, 0)$

C $(4, 0)$ D $(0, 4)$

To find y-intercept we find $f(0)$

$$f(0) = 3(0)^3 + 2(0) - 4 = 0 + 0 - 4 = -4$$

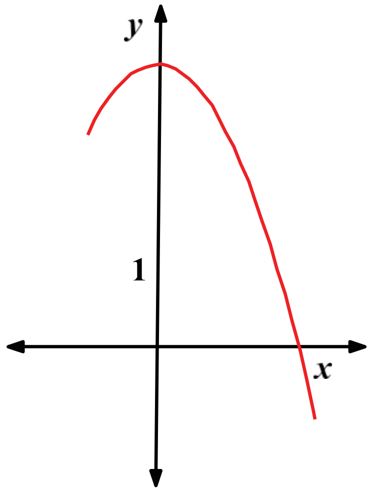
Note:

At x-intercept $y = 0$

At y-intercept $x = 0$

A

13.7 Find the y-intercept



A $(0, 4)$ B $(2, 0)$

C $(4, 0)$ D $(0, 2)$

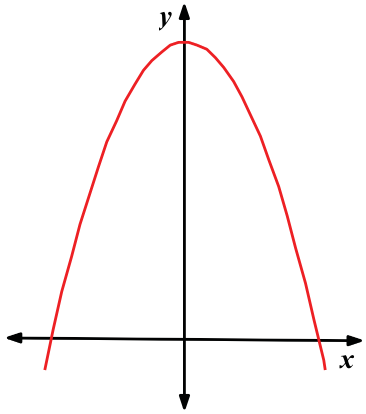
The point at which the graph crosses the y-axis is 4, so the point is $(0, 4)$

A

Even and Odd Functions

Even Function	Functions that are symmetric with respect to the y-axis	$f(-x) = f(x)$
Odd Function	Functions that are symmetric with respect to the ***origin***	$f(-x) = -f(x)$

13.8 Analyze the graph to determine the type of the function.



A Odd B Even

C Neither D It is not a function

The function is symmetric with respect to y-axis therefore it is even function.

B

Note:

$f(-x) = f(x)$ if $f(x)$ is equal to:

x^{even}, $|x|$ or $\cos x$ $\rightarrow$ even functions

$f(x) = -f(x)$ if $f(x)$ is equal to:

x^{odd}, $\tan x$ or $\sin x$ $\rightarrow$ odd functions

13.9 Determine the type of the function

$f(x) = x^6 + 2x^4 + 3x^2 + 5$

A Odd

B Even

C Neither

D Both, odd and even

The constant 5 can be written as $5x^0$. Hence all the exponents of the variable x are even $\rightarrow$ it is an even function.

B

13.10 Determine the type of the function

$f(x) = 3x^5 + 2x^3 + 7x + 4$

A Odd

B Even

C Neither

D Both, odd and even

The constant 4 can be written as $4x^0$. Hence the exponents are both even and odd $\rightarrow$ neither

C

13.11 Determine the odd function

A $2x^7 + x^4$

B $x^5 + 1$

C $f(x) = x^5 + \tan x$

D $f(x) = x^4 + |x|$

The odd functions should have

x^{odd}, $\tan x$ or $\sin x \rightarrow f(x) = x^5 + \tan x$

C

13.12 Determine the even function

A $f(x) = 3x^4 + \cos x$

B $4x^3 + \sin x$

C $f(x) = 4x^6 + \sin x$

D $6x^2 + \tan x + |x|$

The even functions should have x^{even}, $|x|$ or $\cos x$

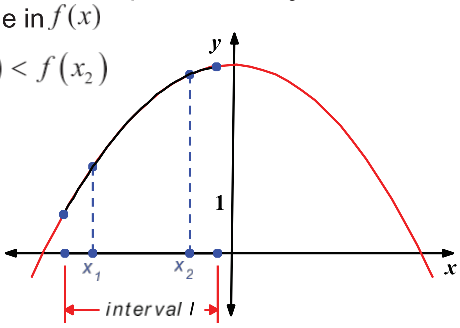
$\rightarrow 3x^4 + \cos x$

A

Increasing, Decreasing and Constant Functions

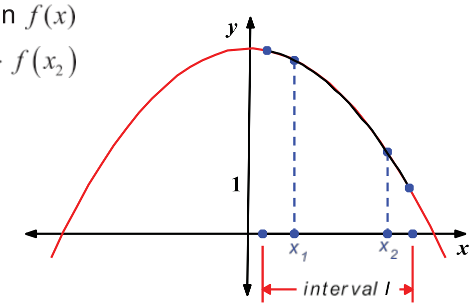
A Function f is **increasing** on an interval I if and only if for any two points in interval I, a positive change in x results in a positive change in $f(x)$

$x_1 < x_2 \rightarrow f(x_1) < f(x_2)$



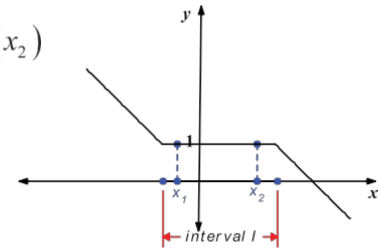
A Function f is decreasing on an interval I if and only if for any two points in I a positive change in x results in a negative change in $f(x)$

$x_1 < x_2 \rightarrow f(x_1) > f(x_2)$

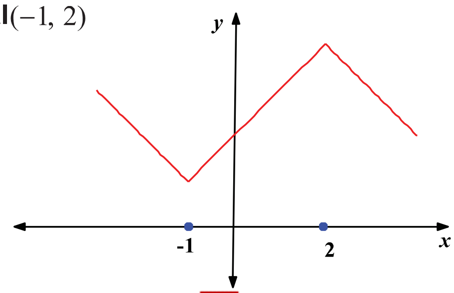


A Function f is constant on an interval I if and only if for any two points in I, a positive change in x results in a zero change in $f(x)$

$x_1 < x_2 \rightarrow f(x_1) = f(x_2)$



13.13 Use the graph to describe the function on the interval $(-1, 2)$



A Increasing

B Decreasing

C Constant

D Neither

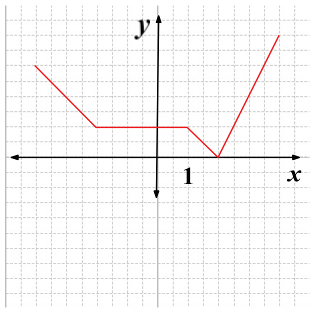
$-1 < 2$

$f(-1) < f(2)$

the function is increasing

A

13.14 Determine the interval on which the function increases



A $(-2, 1)$

B $(-4, -2)$

C $(1, 2)$

D $(2, 4)$

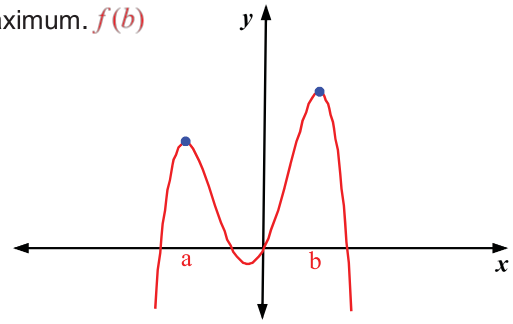
From left to right, the graph increases on the interval $(2, 4)$

D

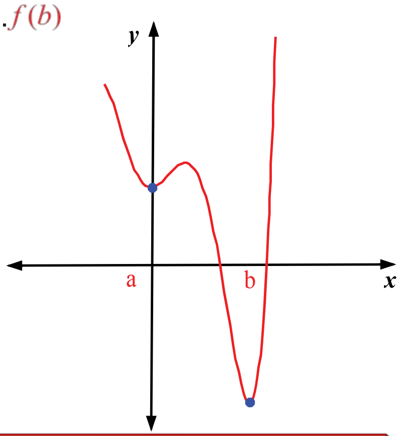
Relative and Absolute Extrema

A relative maximum of a function f is the greatest value $f(x)$ can attain on some interval of the domain. $f(a)$

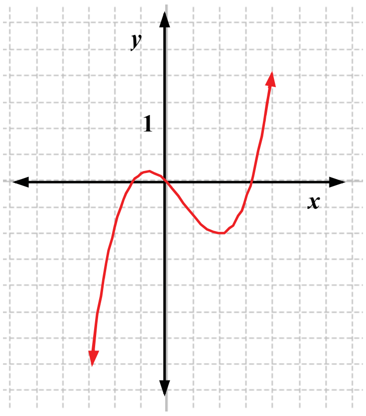
→ If the relative maximum is the greatest value a function can attain over it is entire domain, then it is the absolute maximum. $f(b)$



A relative minimum of a function *f* is the least value $f(x)$ can attain on some interval of the domain. $f(a)$ → If the relative minimum is the least value a function *f* can attain over its entire domain, then it is the absolute minimum. $f(b)$



13.15 Find the relative minimum of function



A $-\infty$ **B** $+\infty$

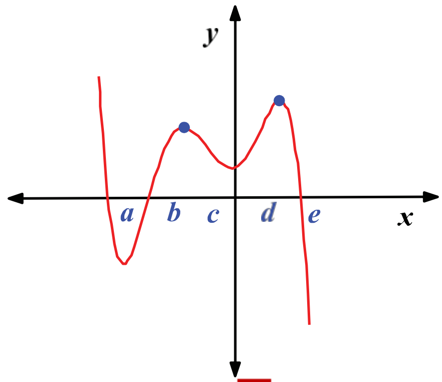
C 1 **D** -1

The relative minimum is -1 since it is the least value of $f(x)$ in the interval $(0,2)$

➤ **D**

Note:
This function does not have absolute maximum or minimum

13.16 The value of $f(x)$ at $x=b$ is a .....



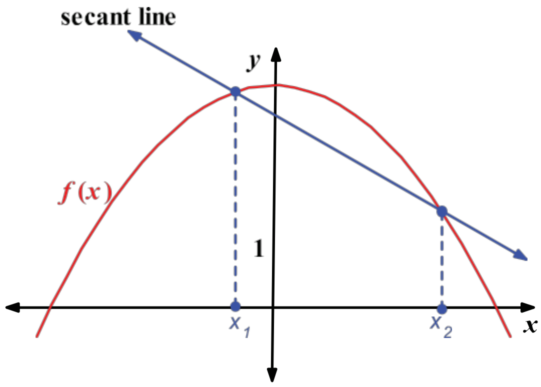
A Relative maximum **B** Relative minimum

C Absolute maximum **D** Absolute minimum

$f(b)$ is the greatest value of the function in the interval (a,c) but we can find $f(d)>f(b)$ in the interval (a,e) Therefore $f(b)$ is a relative maximum.

➤ **C**

Average Rate of Change



→ The **average rate** of change between any two points on the graph of *f* is the slope of the line through those points.

→ The line through the two points on a curve is called a **secant** line. The slope of the secant line is denoted m_{sec}

→ The average rate of change on the interval $[x_1, x_2]$ is

$$m_{sec}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

13.17 Find the average rate of $f(x)=-x^3+3x$ on the interval $[-2,-1]$

A 4 **B** 0

C -4 **D** -2

$$Average\ rate=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$=\frac{f(-1)-f(-2)}{-1-(-2)}$$

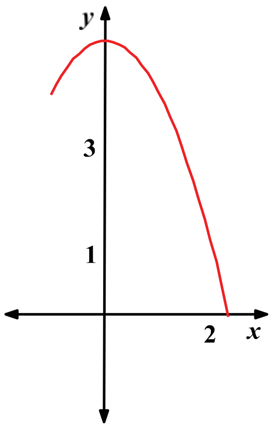
$$=\frac{\left[-(-1)^3+3(-1)\right]-\left[-(-2)^3+3(-2)\right]}{-1+2}$$

$$=\frac{-2-2}{1}$$

$$=-4$$

➤ **C**

13.18 Use the graph of $f(x)$ to find the average rate of change on the interval $[0, 2]$



A -2 **B** 2

C 4 **D** -4

$$Average\ rate = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(2) - f(0)}{2 - 0}$$
$$= \frac{1 - 5}{2}$$
$$= -2$$

➢ **A**

13.19 The height of an object that is thrown from a straight up from a height above ground is given by $h(t) = -16t^2 - 4,$ where t is the time in seconds after the object is thrown. Find the average speed of the object from 0 to 2 seconds.

A 32 **B** -32

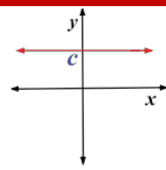
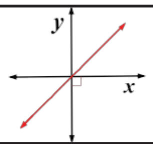
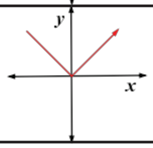
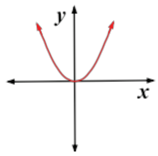
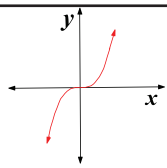
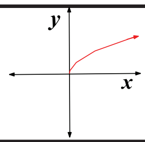
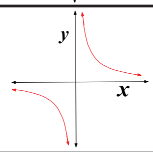
C 64 **D** 0

$$m_{sec} = \frac{f(2) - f(0)}{2 - 0}$$
$$= \frac{\left[-16(2)^2 - 4\right] - \left[-16(0)^2 - 4\right]}{2}$$
$$= \frac{\left[-64 - 4\right] - \left[-4\right]}{2}$$
$$= \frac{-64}{2}$$
$$= -32$$

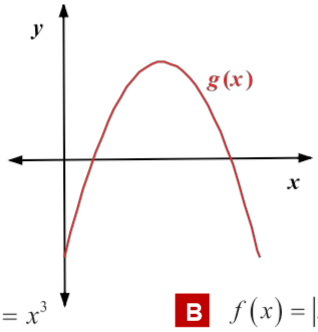
➢ **B**

Parent Functions

A family of functions is a group of functions with graphs that display one or more similar characteristics. Apparent function is the simplest of the function in a family. This is the function that is transformed to create other numbers in a family of functions.

Linear and polynomial parent Functions		
1	A constant function has the form $f(x) = c$ where c is any real number. Its graph is horizontal line. When $c = 0$, $f(x)$ is the zero function.	
2	The Identity function $f(x) = x$ passes through all points with coordinates (a, a).	
3	The absolute value function denoted $f(x) = \lvert x \rvert$ is a V-**shaped** function.	
4	The quadratic function $f(x) = x^2$ has U-**shaped** graph.	
5	The cubic function $f(x) = x^3$ is symmetric about the origin.	
6	The square root function has the form $f(x) = \sqrt{x}$	
7	The reciprocal function has the form $f(x) = \frac{1}{x}$	

13.20 What is the parent function of $g(x)$?



A $f(x)=x^3$ **B** $f(x)=|x|$

C $f(x)=x^2$ **D** $f(x)=\frac{1}{x}$

This function is U-**shaped** but flipped

$\rightarrow f(x)=x^2$

C

13.21 Find the parent function of the function $h(x)=(x-4)^3+9$

A $f(x)=x$ **B** $f(x)=x^2$

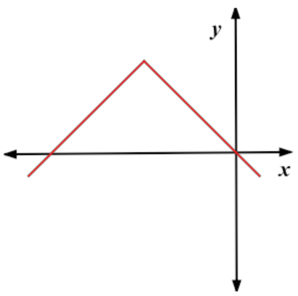
C $f(x)=x^3$ **D** $f(x)=x^4$

The greatest exponent of the function $h(x)$ is 3, therefore it is a cubic function

$\rightarrow f(x)=x^3$

C

13.22 Find the parent function of the function $g(x)$



A $f(x)=|x+2|$ **B** $f(x)=|x|$

C $f(x)=|x-2|$ **D** $f(x)=|x|+2$

The function $g(x)$ has V-**shaped** therefore it is an absolute value function $f(x)=|x|$

B

Transformation of a Function

- Vertical translation (up or down)
- ***k*** is outside the function
  - Translation up ***k*** units, $y=f(x)+k$
  - Translation down ***k*** units, $y=f(x)-k$
- Horizontal translation (left or right)
- ***h*** is inside the function
  - Translation right ***h*** units, $y=f(x-h)$
  - Translation left (***h***) units, $y=f(x+h)$
- Reflections
  - In the **x-axis** $y=-f(x)$
  - In the **y-axis** $y=f(-x)$

13.23 What is the equation of $y=x^2$ translated 5 units up and 3 units left.

A $f(x)=(x-5)^2+3$ **B** $f(x)=(x+3)^2+5$

C $f(x)=(x-3)^2-5$ **D** $f(x)=(x-5)^2+3$

Combined transformation $f(x-h)+k$

h represents left or right $(x-(-3))^2=(x+3)^2$

k represents up or down $(x+3)^2+5$

B

13.24 Find the horizontal translation to the function $h(x)=|x+4|-2$

A 4 **B** -4

C 2 **D** -2

$$h(x)=f(x-h)+k$$
$$=|x+4|+(-2)$$
$$=|x-(-4)|-2$$
$$h=-4$$

B

13.25 Determine the value and direction of the transformation $f(x)=\sqrt{x-4}$

A Four units, left **B** Four units, right

C Four units, up **D** Four units, down

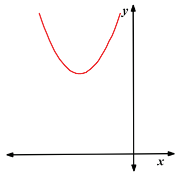
$f(x-h)+k$

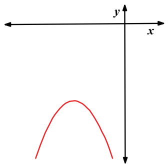
h represents the horizontal translation

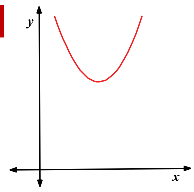
$h=4 \rightarrow h>0$ translate to the right

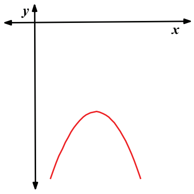
B

13.26 Which of the following is the graph of
$y=(x+2)^2+3$

A


B


C


D


Compare $=(x+2)^2+3$ to $(x-h)^2+k$

$h=-2 \rightarrow$ Two units, left

$k=3 \rightarrow$ Three units, up

➢ **A**

13.27 Write an equation for the transformation of the graph of $f(x)=|x|$ to $g(x)$ translated 2 units up, 3 units right and reflected in the x-axis

A $g(x)=-|x-3|+2$ **B** $g(x)=|x+3|+2$
C $g(x)=-|x-3|-2$ **D** $g(x)=|-x-3|-2$

Reflect in the ***x*-axis** $y=-|x|$

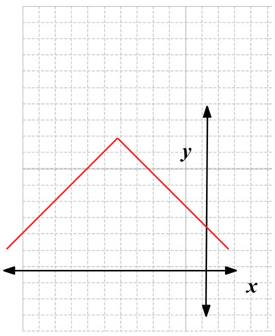
Compare $y=|x-h|+k$ to find hand k

$h=3, \quad k=2$

$y=g(x)=-|x-3|+2$

➢ **A**

13.28 Use the parent function of $f(x)=|x|$ to write the equation of $g(x)$

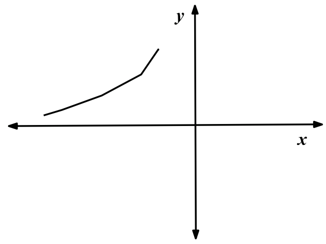


A $g(x)=-|x+3|+2$ **B** $g(x)=-|x-3|+2$
C $g(x)=-|x+2|+3$ **D** $g(x)=-|x-2|+3$

Trace the graph

➢ **C**

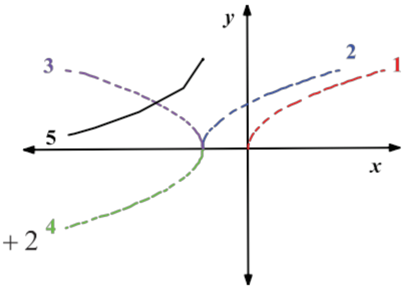
13.29 Use the parent function of $f(x)=\sqrt{x}$ to write the equation of $g(x)$



A $g(x)=-\sqrt{-(x-1)}+2$ **B** $g(x)=-\sqrt{-(x+1)}+2$
C $g(x)=-\sqrt{x-1}+2$ **D** $g(x)=-\sqrt{x+1}+2$

Trace the graph of the parent function

1) $f(x)=\sqrt{x}$
2) $h(x)=\sqrt{x+3}$
3) $m(x)=\sqrt{-(x+3)}$
4) $n(x)=-\sqrt{-(x+3)}$
5) $g(x)=-\sqrt{-(x+3)}+2$



➢ **B**