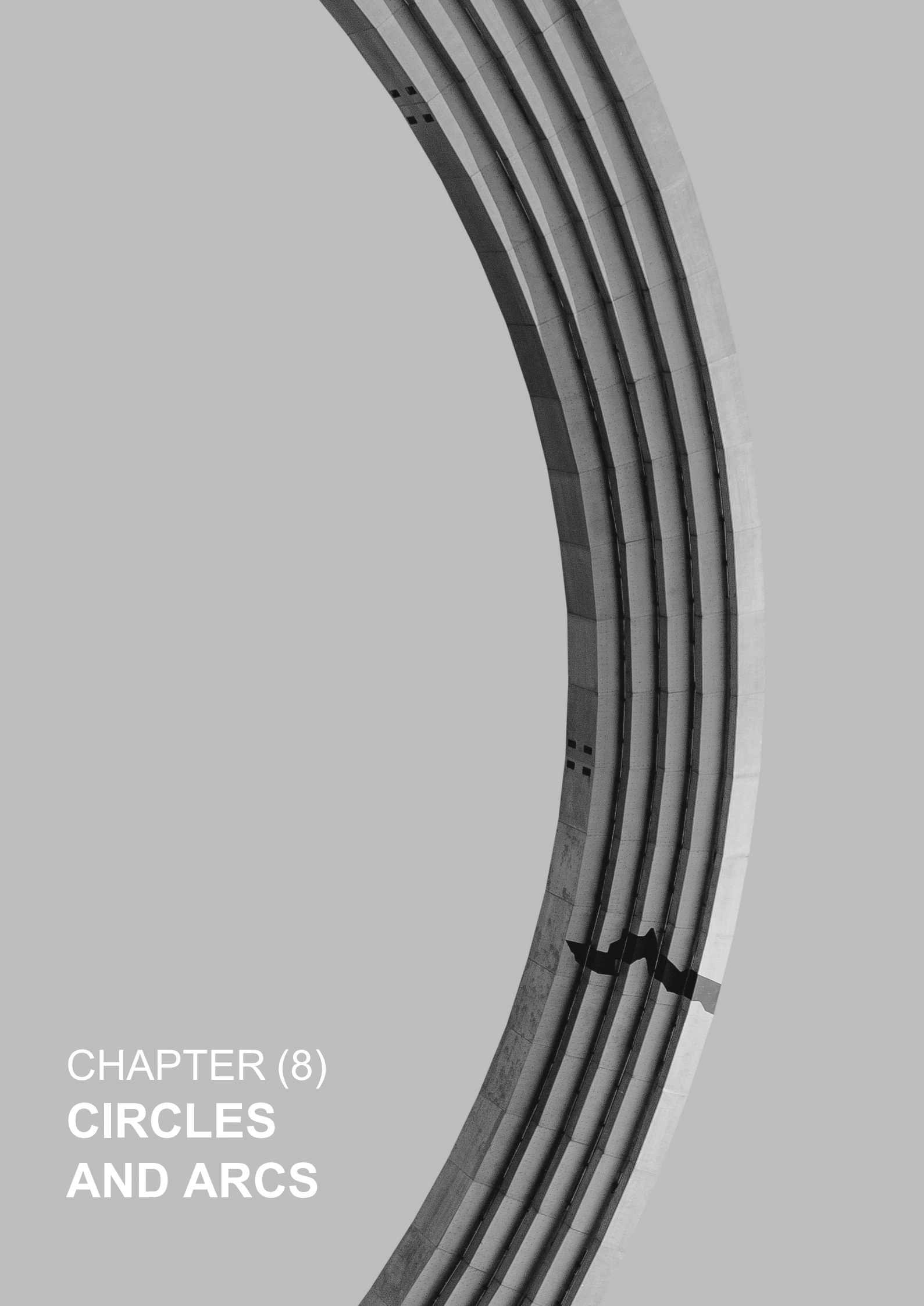
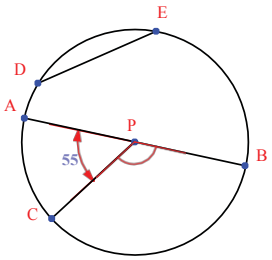
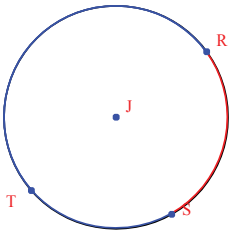


CHAPTER (8)

CIRCLES AND ARCS

- Circle is the set of all points equidistant from a given point called the center (P)
- A diameter is a segment that contains the center of the circle and has both end points on the circle. $\overline{AB}$
- A radius is segment that has one end point at the center and the other end point on the circle. $\overline{BC}$
- Diameter is twice the radius $d = 2r$

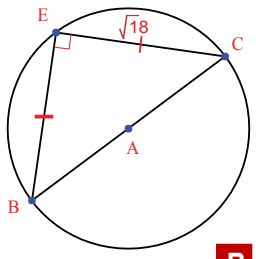




- A central angel is an angle whose vertex is the center of the circle. $\angle CPB$
- An arc is a part of a circle.
- Semicircle is half of a circle $(180°)$
- A minor arc is smaller than a semicircle $(<180°)$ $\overset{\frown}{RS}$
- The measure of the minor arc is equal to the measures of the corresponding central angle $m\overset{\frown}{AC} = m\angle APC = 55°$
- A major are is larger than a semicircle $(>180°)$ $\overset{\frown}{STR}$
- The measure of the major arc is equal to the measures of the related minor arc subtracted from 360° $m\overset{\frown}{CBA} = 360 - m\overset{\frown}{AC} = 305°$
- The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs $m\overset{\frown}{ACB} = m\angle\overset{\frown}{AC} + m\angle\overset{\frown}{CB}$
- A chord is a segment that has two end points on the circle $\overline{DE}$
- In a circle: Congruent Chords ↔ Congruent Arcs
- The sum of non over lapping central angles is 360°

	In terms of diameter	In terms of radius
Circumference of a circle	$C = d\pi$	$C = 2\pi r$
Area of a circle		$A = \pi r^2$

1. Find the circumference of the circle



A 6 **B** 6π

C 18 **D** 18π

$45°\text{-}45°\text{-}90° \rightarrow hypo = L\sqrt{2}$

$= \sqrt{18}\cdot\sqrt{2}$

$= \sqrt{36}$

$= 6$

$C = d\pi$

$= 6\pi$ ➤ **B**

2. Find the radius of the circle if its circumference is $50\ m$

A 6 **B** 7

C 8 **D** 9

$C = 2\pi r$

$50 = 2\pi r$

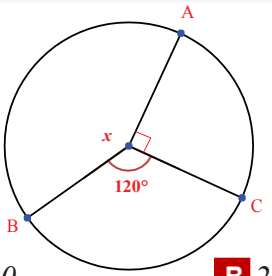
$r = \dfrac{50}{2\pi}$

$= \dfrac{25}{\pi}$

$= \dfrac{25}{3.14}$

≈ 8 ➤ **C**

3. Find x



A 140 **B** 210

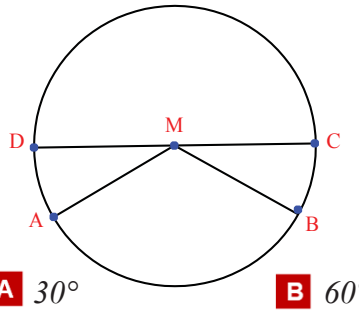
C 150 **D** 120

$x + 90 + 120 = 360$

$x + 210 = 360$

$x = 150$ ➤ **C**

4. If $m\overset{\frown}{AB} = 4m\overset{\frown}{BC}$ and $m\overset{\frown}{BC} = m\overset{\frown}{AD}$, Then find $m\overset{\frown}{AD}$

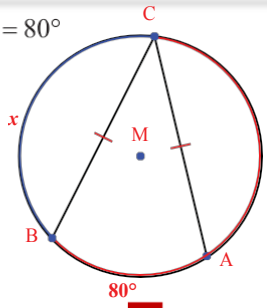


A $30°$ **B** $60°$
C $45°$ **D** $25°$

The semicircle $\overset{\frown}{DAC}$ contains the arcs $\overset{\frown}{DA}$, $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$

$$m\overset{\frown}{AD} + m\overset{\frown}{AB} + m\overset{\frown}{BC} = 180^0$$
$$m\overset{\frown}{AD} + 4m\overset{\frown}{BC} + m\overset{\frown}{BC} = 180^0$$
$$m\overset{\frown}{AD} + 5m\overset{\frown}{BC} = 180^0$$
$$6m\overset{\frown}{AD} = 180° \qquad m\overset{\frown}{AD} = m\overset{\frown}{BC}$$
$$m\overset{\frown}{AD} = 30° \Rightarrow \textbf{A}$$

5. Find x if $m\overset{\frown}{AB} = 80°$

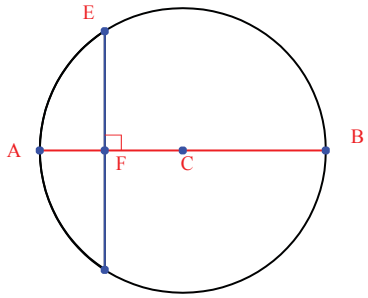


A 40 **B** 80
C 140 **D** 150

Congruent chords ↔ Congruent arcs

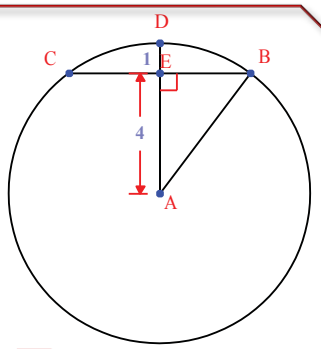
$$m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CA} = 360°$$
$$80 + x + x = 360°$$
$$2x + 80 = 360°$$
$$2x = 280°$$
$$x = 140° \Rightarrow \textbf{C}$$

Theorem: In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and vice versa



If $\overline{AB}$ is a diameter and $\overline{EF} \cong \overline{FG}$, ↔ $\overline{AB} \perp \overline{EG}$

6. Find the length of CB



A 3 **B** 6
C 4 **D** 8

the radius is $1 + 4 = 5 \rightarrow AB = 5$

Using Pythagorean triple $(3, 4, 5)$
the third EB side is 3

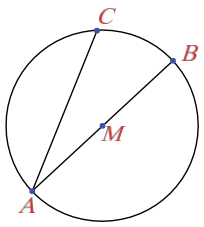
Since the perpendicular radius bisects the chord then $CE = EB$

$$CB = CE + EB$$
$$= 3 + 3 = 6 \Rightarrow \textbf{B}$$

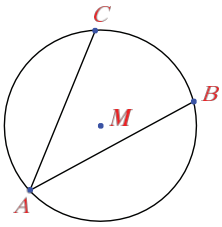
Inscribed Angle

Inscribed angle is an angle whose vertex is on the circle and whose sides are chords of the circle

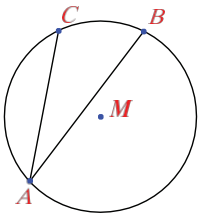
There are three cases to consider



The center is on the side of the angle

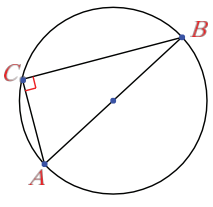


The center is inside the angle

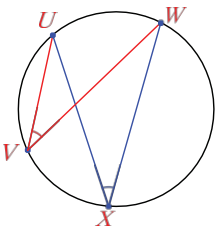


The center is outside the angle

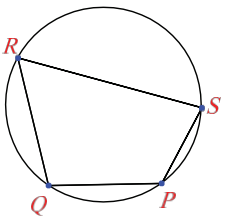
Inscribed Angle Theorem



An angle inscribed in a semicircle is a right angle



Two inscribed angles that intercept the same are congruent

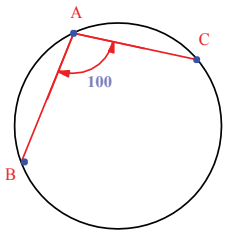


The opposite angles of a quadrilateral inscribed in a circle are supplementary.

The measure of an inscribed angel is half

$$m\angle V = \frac{1}{2} m\overset{\frown}{UV}$$

7. Find the measure of the arc $\overset{\frown}{CB}$



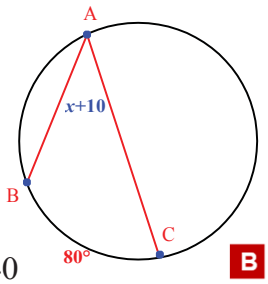
A 100° **B** 150°
C 180° **D** 200°

Since $\overset{\frown}{CAB}$ is an inscribed angle $m\angle A = \frac{1}{2} m\overset{\frown}{CB}$

$$100° = \frac{1}{2} m\overset{\frown}{CB}$$

$$200° = m\overset{\frown}{CB}$$ ➤ **D**

8. What is the value of x?



A 40 **B** 30
C 80 **D** 50

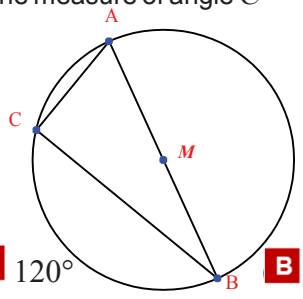
$$m\angle A = \frac{1}{2} m\overset{\frown}{CB}$$

$$x + 10 = \frac{1}{2} \quad 80$$

$$x + 10 = 40$$

$$x = 30$$ ➤ **B**

9. Find the measure of angle C

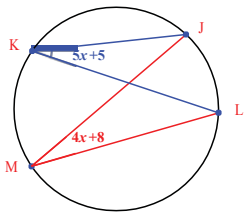


A 120° **B** 80°
C 90° **D** 60°

The angle inscribed in a semicircle is a right angle $m\angle C = 90°$

➤ **C**

10. What is the value of x



A 3 **B** 4
C 20 **D** 18

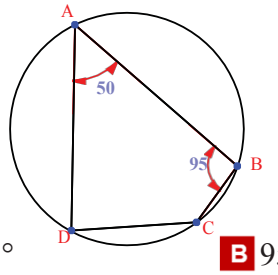
The two angles intercept the same arc; therefore, the angles are congruent.

$$4x + 8 = 5x + 5$$

$$8 - 5 = 5x - 4x$$

$$x = 3$$ ➤ **A**

11. Find the measure of angle D

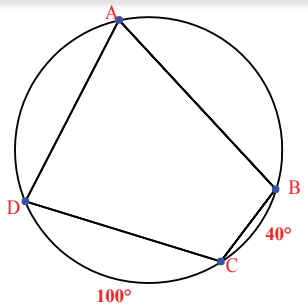


A 85° **B** 95°
C 100° **D** 105°

$$m\angle 95 + m\angle D = 180°$$

$$m\angle D = 85°$$ ➤ **A**

12. Find $m\angle C$



A 70° **B** 80°
C 140° **D** 110°

$$m\overset{\frown}{BCD} = m\overset{\frown}{BC} + m\overset{\frown}{CD}$$

$$= 40 + 100$$

$$= 140°$$

$$m\angle A = \frac{1}{2} m\overset{\frown}{BCD}$$

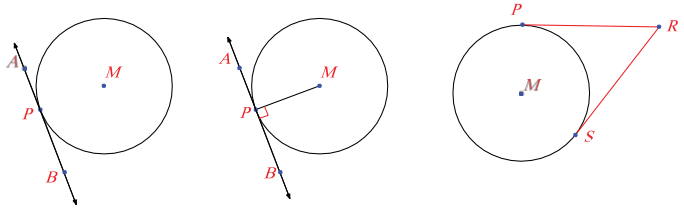
$$= \frac{1}{2} \cdot 140 = 70°$$

$$m\angle A + m\angle C = 180°$$

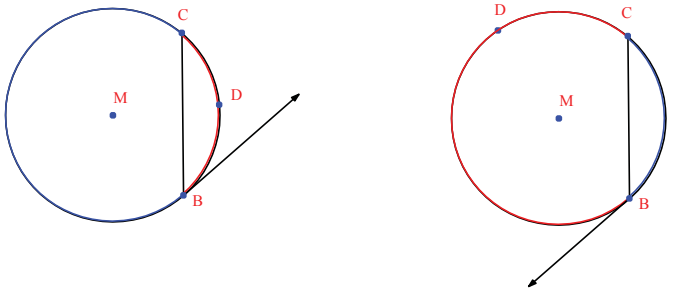
$$70 + m\angle C = 180°$$

$$m\angle C = 110°$$ ➤ **D**

Tangent Lines



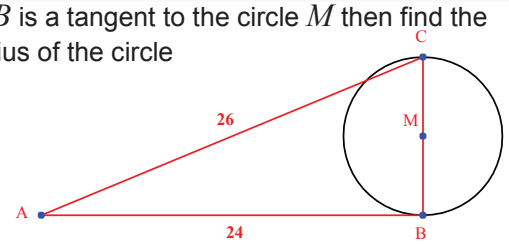
- A tangent to a circle is a line in the plane of the circle that intersects the circle in exactly one point.
- The point where a circle and a tangent intersect is the point of tangency.
- If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.
- If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.



- The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc

$$m\angle B = \frac{1}{2}\overset{\frown}{BDC}$$

13. If AB is a tangent to the circle M then find the radius of the circle



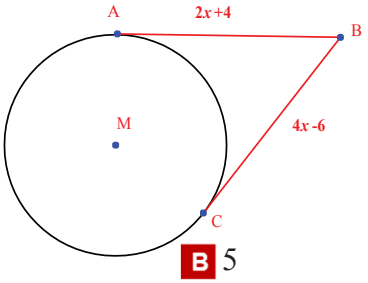
A 10 **B** 100
C 5 **D** 20

The tangent is perpendicular to the diameter $\overline{CB} \perp \overline{AB}$

ΔABC is a right triangle and by the Pythagorean triple $2(5,12,13) \rightarrow (10,24,26) \rightarrow CB = 10$

$\overline{CB}$ is the diameter of the circle: $r = \frac{d}{2} = 5$ ➪ **C**

14. What is the value of x if $\overline{AB}$ and $\overline{BC}$ are two tangents to the circle M

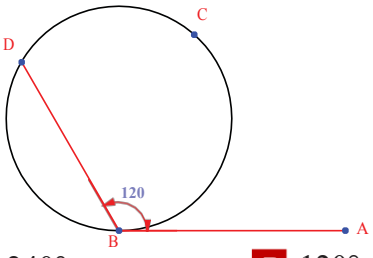


A 10 **B** 5
C 4 **D** 2

The two tangents are congruent

$$2x+4 = 4x-6$$
$$10 = 2x$$
$$x = 5$$ ➪ **B**

15. Find the $m\overset{\frown}{BCD}$ if $m\angle ABD = 120°$ and AB is a tangent to circle

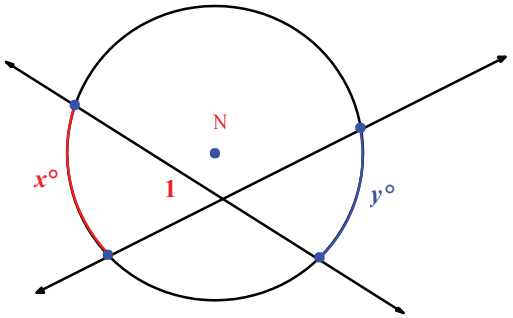


A 240° **B** 120°
C 30° **D** 60°

$$m\angle B = \frac{1}{2}m\overset{\frown}{BCD}$$
$$m\overset{\frown}{BCD} = 2m\angle B$$
$$= 2\cdot 120 = 240°$$ ➪ **A**

Angle Measures and Segment Lengths

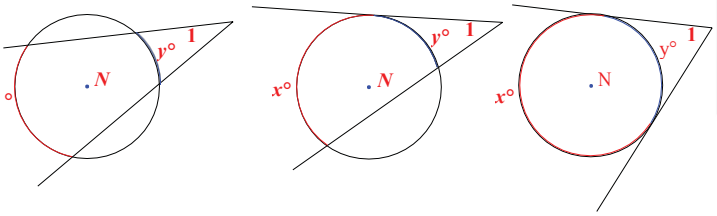
1- The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted *arcs*.



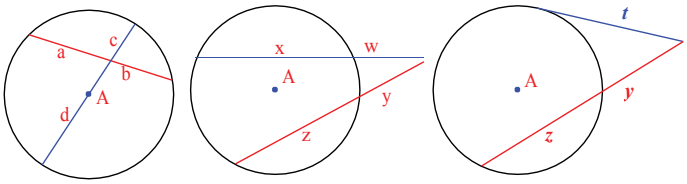
$$m\angle 1 = \frac{1}{2}(x+y)$$

2- The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted *arcs*

$$m\angle 1 = \frac{1}{2}(x - y)$$

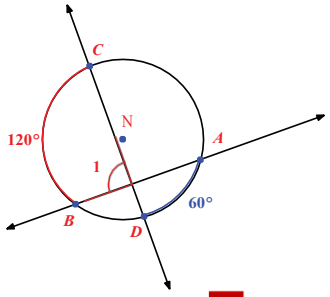


3 - For a given point and circle, the product of the lengths of two segments from the point to the circle is constant a long any line through the point and circle



$a \cdot b = c \cdot d$ $\quad (w+x)w = (y+z)y$ $\quad (y+z)y = t^2$

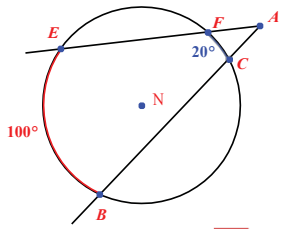
16. Find $m\angle 1$



A 180° **B** 90°
C 120° **D** 240°

$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CB} + m\overset{\frown}{AD})$$
$$= \frac{1}{2}(120 + 60)$$
$$= \frac{1}{2}(180) = 90° \Rightarrow \textbf{B}$$

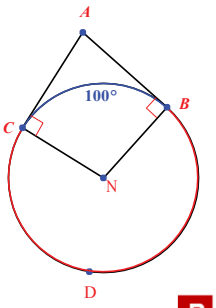
17. Find $m\angle A$



A 30° **B** 80°
C 10° **D** 40°

$$m\angle A = \frac{1}{2}(m\overset{\frown}{EB} - m\overset{\frown}{CF})$$
$$= \frac{1}{2}(100 - 20)$$
$$= \frac{1}{2}(80) = 40° \Rightarrow \textbf{D}$$

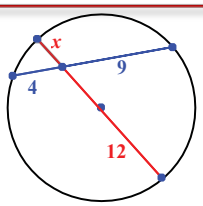
18. Find the measure of angle A



A 160° **B** 80°
C 120° **D** 50°

$$m\overset{\frown}{BDC} = 360 - m\overset{\frown}{BC}$$
$$= 360 - 100 = 260$$
$$m\angle A = \frac{1}{2}\left(m\overset{\frown}{BDC} - m\overset{\frown}{BC}\right)$$
$$= \frac{1}{2}(260 - 100)$$
$$= \frac{1}{2}(160) = 80° \Rightarrow \textbf{B}$$

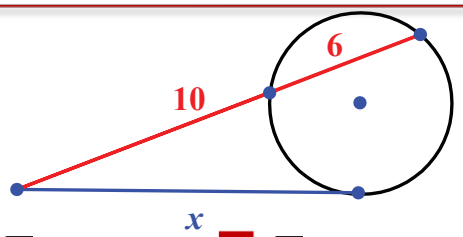
19. Find x



A 3 **B** 36
C 18 **D** 6

$$a \cdot b = c \cdot d$$
$$12x = 4 \cdot 9$$
$$12x = 36$$
$$x = 3 \Rightarrow \textbf{A}$$

20. Find x

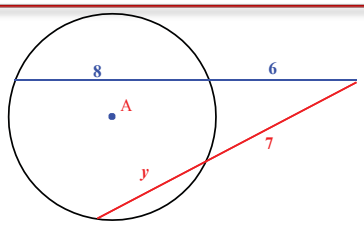


A $4\sqrt{10}$ B $\sqrt{10}$

C 160 D 40

$$t^2 = (y + z)\,y$$
$$x^2 = (10+6)\,10$$
$$x^2 = 16\times 10$$
$$x = \sqrt{16\times 10}$$
$$x = 4\sqrt{10}$$ ➤ A

21. Find y



A 35 B 5

C 49 D 30

$$(w+x)\,w = (y + z)\,y$$
$$(6+8)\,6 = (y + 7)\,7$$
$$84 = 49 + 7y$$
$$35 = 7y$$
$$y = 5$$ ➤ B

Equation of a Circle

- An equation of a circle with center (h,k) and radius r is $(x-h)^2+(y-k)^2=r^2$
- If the center is $(0,0)$ then the equation is $x^2+y^2=r^2$

22. Find the center of the circle $(x+7)^2+(y-5)^2=16$

A $(-7,5)$ B $(7,-5)$

C $(5,-7)$ D $(-5,\ 7)$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-7))^2+(y-5)^2=16$$
$$\rightarrow h=-7$$
$$\rightarrow k=5$$

Center $(-7,\ 5)$ ➤ A

23. Find the diameter of the circle

$$(x-4)^2+(y-1)^2=25$$

A 10 B 25

C 8 D 2

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-4)^2+(y-1)^2=25$$
$$\rightarrow r^2=25$$
$$r=5$$
$$d=2r$$
$$=2\cdot 5$$
$$=10$$ ➤ A