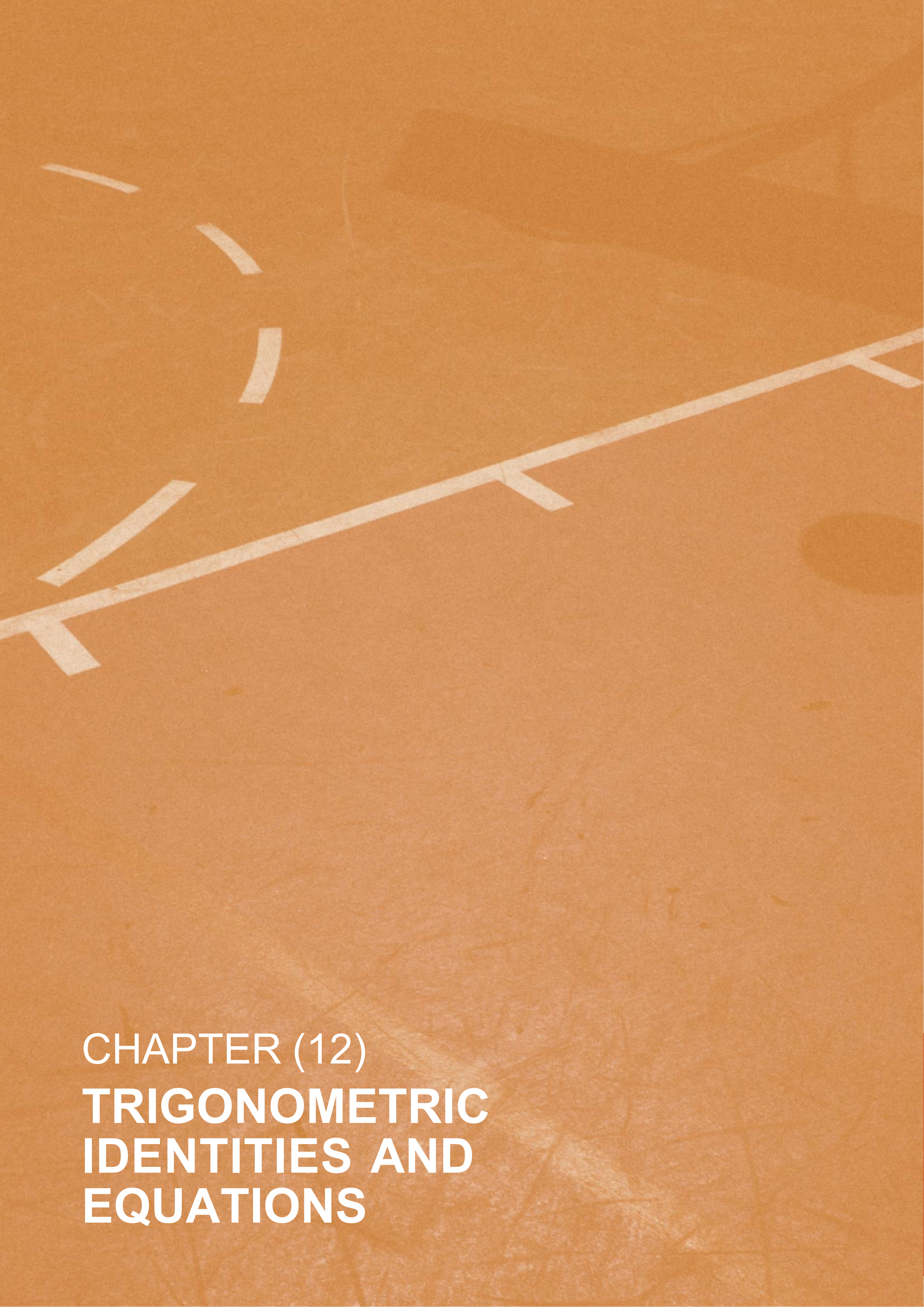


CHAPTER (12)

TRIGONOMETRIC IDENTITIES AND EQUATIONS

Law of Sines

In any triangle the ratio of the sine of each angle to its opposite side is constant

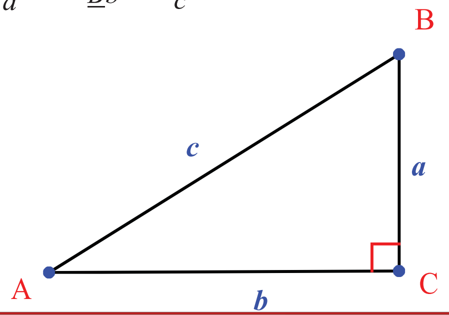
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines

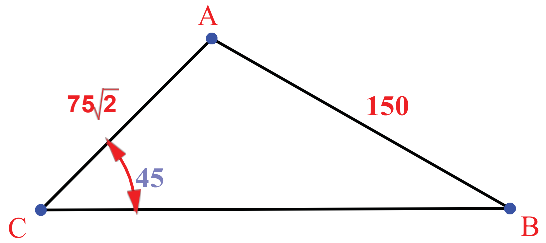
In any triangle

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = b^2 + a^2 - 2ab \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



12.1 Find the measure of angle B



A $30°$ **B** $45°$

C $15°$ **D** $60°$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{75\sqrt{2}} = \frac{\sin 45}{150}$$

$$\frac{\sin B}{75\sqrt{2}} = \frac{\frac{\sqrt{2}}{2}}{150}$$

$$\sin B = \frac{75 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2}}{150}$$

$$= \frac{75 \ \frac{2}{2}}{150}$$

$$= \frac{75}{150}$$

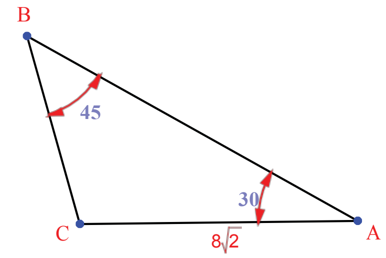
$$= \frac{1}{2}$$

$$\sin^{-1} \frac{1}{2} = B$$

$$m\angle B = 30°$$

A

12.2 Find the length of BC



A 4 **B** $8\sqrt{2}$

C 8 **D** 10

$$\frac{\sin A}{BC} = \frac{\sin B}{AC}$$

$$\frac{\sin 30}{BC} = \frac{\sin 45}{8\sqrt{2}}$$

$$BC = \frac{8\sqrt{2} \sin 30}{\sin 45}$$

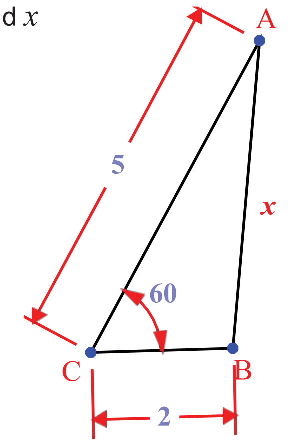
$$= \frac{8\sqrt{2} \cdot \frac{1}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{4\sqrt{2}}{\frac{\sqrt{2}}{2}}$$

$$= 4\sqrt{2} \cdot \frac{2}{\sqrt{2}} = 8$$

C

12.3 Find x



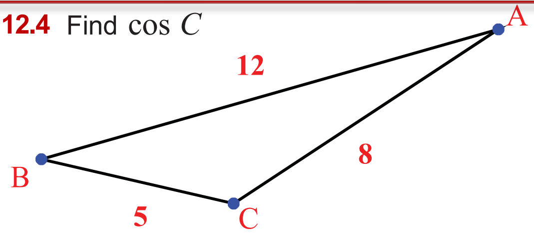
A 19 **B** $\sqrt{19}$

C 9 **D** $\sqrt{9}$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 5^2 + 2^2 - 2 \cdot 2 \cdot 5 \cos 60$$
$$= 25 + 4 - 20 \cdot \frac{1}{2}$$
$$= 29 - 10$$
$$c^2 = 19$$
$$c = \sqrt{19}$$

B

12.4 Find $\cos C$



A $-\dfrac{55}{80}$ B $\dfrac{55}{80}$

C $\dfrac{233}{80}$ D $\dfrac{-233}{80}$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$12^2 = 5^2 + 8^2 - 2\cdot 5\cdot 8\cos C$$

$$144 = 25 + 64 - 80\cos C$$

$$55 = -80\cos C$$

$$\cos C = \frac{-55}{80}$$

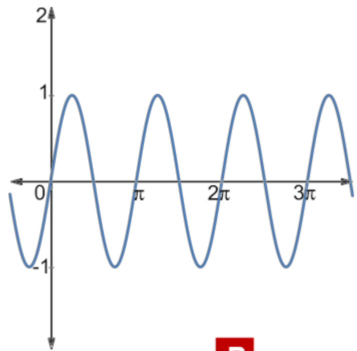
Or use directly $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ ➢A

Analyse the Periodic Functions

Parent function $a = 1,\ b = 1$			
Function	sinθ	cosθ	tanθ
Period	2π, 360^0	2π, 360^0	,180
Amplitude	1	1	undefined

$a \neq 0$, $b > 0$			
Function	$a\sin b\theta$	$a\cos b\theta$	$a\tan b\theta$
Period	$\dfrac{2\pi}{b}$, $\dfrac{360^0}{b}$	$\dfrac{2\pi}{b}$, $\dfrac{360^0}{b}$	$\dfrac{\pi}{b}$, $\dfrac{180^0}{b}$
Amplitude	$\lvert a\rvert$	$\lvert a\rvert$	undefined

12.5 Analyze the periodic function to find its period.



A $\dfrac{\pi}{2}$ B π

C 2π D 4π

One cycle begins at $x = 0$ and ends at $x = \pi$

$\pi - 0 = \pi$, so the period of the function is π ➢A

12.6 Find the period and the amplitude of the function $y = 4\cos 5\theta$

A $5,\ 90^0$ B $4,\ 72^0$

C $4,\ 50^0$ D $4,\ 36^0$

$$\text{amplitude} = |a| = |4| = 4$$

$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{5}$$

$$= \frac{360}{5} = 72^\circ$$

➢B

12.7 Which function has amplitude = 4 and period = $\dfrac{\pi}{2}$

A $y = 4\cos 4\theta$ B $4\tan 4\theta$

C $y = 4\sin 2\theta$ D $y = 2\cos 4\theta$

The amplitude of $\tan\theta$ is undefined; therefore, option B is excluded

So, only we will check $a\sin b\theta$ or $a\cos b\theta$

where $|a|$ is amplitude =4

A) $|-4| = 4$ C) $|4| = 4$

$$\frac{\pi}{2} = 90^0$$

Period in cos and sine = $\dfrac{360}{b} \rightarrow \dfrac{360}{4} = 90$

Therefore, option A has period 90

➢A

12.8 Simplify $\dfrac{\cot\theta\ \tan\theta}{\dfrac{\cos\theta}{\sin\theta}}$

A $\cot\theta$ B $\tan\theta$

C $\sin\theta$ D $\cos\theta$

Recall that $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$

$$\frac{\cot\theta\tan\theta}{\dfrac{\cos\theta}{\sin\theta}} = \frac{\cot\theta\tan\theta}{\cot\theta}$$

$$= \tan\theta$$

➢B

Angle Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$\div \sin^2\theta \downarrow$ $\qquad$ $\div \cos^2\theta \downarrow$

$$1+\cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

Negative Angle Identities	Co-function Identities
$\sin(-\theta) = -\sin\theta$	$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$
$\cos(-\theta) = \cos\theta$	$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$
$\tan(-\theta) = -\tan\theta$	$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

Angle Sum and Difference Identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Half Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Double Angle Identities

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan\ \theta}$$

12.9 Simplify $\tan^2\theta\cos^2\theta =$

A $\sin\theta$ $\qquad$ **B** $\sin^2\theta$

C $\cos\theta$ $\qquad$ **D** $\cos^2\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta\cos^2\theta = \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\ \cancel{\cos^2\theta}$$

$$= \sin^2\theta$$

B

12.10 If $270° < \theta < 360°$, $\sin\theta = \frac{1}{2}$ then find $\cos\theta$

A $\pm\frac{\sqrt{3}}{2}$ $\qquad$ **B** $\frac{1}{2}$

C $\frac{\sqrt{3}}{2}$ $\qquad$ **D** $-\frac{1}{2}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\cos\theta = \pm\sqrt{\frac{3}{4}}$$

$$= \pm\frac{\sqrt{3}}{2}$$

θ is in quadrant IV, $\rightarrow$ $\cos\theta$ is positive

$$\cos\theta = \frac{+\sqrt{3}}{2}$$

C

12.11 Simplify $(1-\cos^2\theta)\sin^2\theta$

A $\cos^4\theta$ B $\sin^4\theta$
C $\sec^4\theta$ D $\csc^4\theta$

$\sin^2\theta + \cos^2 = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\rightarrow (1-\cos^2\theta).\sin^2\theta$

$=\sin^2\theta.\sin^2\theta$

$=\sin^4\theta$ ⮚ B

12.12 Find $\sin\left[\sin^2(\sec 105)\right]+\left[\cos^2(\sec 105)\right]$

A 105 B 60
C $\frac{1}{2}$ D 1

assume $\theta = \sec 105$

$\left[\sin^2(\sec 105)\right]+\left[\cos^2(\sec 105)\right]$

$\sin^2\theta + \cos^2\theta = 1$

$\rightarrow 1$ ⮚ D

12.13 Simplify $\tan^2(\cot^2\theta - \cos^2\theta)$

A $\sin^2\theta$ B $\cos^2\theta$
C $\tan^2\theta$ D $\sin\theta$

$\tan^2\theta(\cot^2\theta - \cos^2\theta) = \tan^2\theta.\cot^2\theta - \tan^2\theta.\cos^2\theta$

$= \tan^2\theta \times \frac{1}{\tan^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$

$= 1 - \sin^2\theta = \cos^2\theta$ ⮚ B

12.14 Find the determinant of $\begin{bmatrix}\sin x & -\cos x\\ \cos x & \sin x\end{bmatrix}$

A $\text{Cos } 2x$ B 0
C 1 D $\sin 2x$

$\sin x\cdot\sin x - (-\cos x)\cdot\cos x$

$\sin^2 x + \cos^2 x = 1$ ⮚ C

12.15 Simplify $\tan\theta\cdot\sin\left(\frac{\pi}{2}-\theta\right)$

A $\sin\theta$ B $\cos\theta$
C $\tan\theta$ D $\cot\theta$

$\tan\theta\cdot\sin\left(\frac{\pi}{2}-\theta\right) = \frac{\sin\theta}{\cos\theta}\cdot\cos\theta$

$= \sin\theta$ ⮚ A

12.16 Find the exact value of $\cos 15^\circ$

A $\frac{\sqrt{6}+\sqrt{2}}{2}$ B $\frac{\sqrt{6}+\sqrt{2}}{4}$
C $\frac{\sqrt{6}-\sqrt{2}}{2}$ D $\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos 15 = \cos(45-30)$

$= \cos 45\cos 30 + \sin 45\sin 30$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ ⮚ B

12.17 Evaluate $\cos(30-\theta)\cos\theta - \sin(30-\theta)\sin\theta$

A $\frac{-\sqrt{3}}{2}$ B $\frac{1}{2}$
C $\frac{-1}{2}$ D $\frac{\sqrt{3}}{2}$

$\cos(30-\theta)\cos\theta - \sin(30-\theta)\sin\theta = \cos[(30-\theta)+\theta]$

$=\cos 30$

$= \frac{\sqrt{3}}{2}$ ⮚ D

12.18 Simplify $\cos^4\theta - \sin^4\theta$

A $\sin 2\theta$ B $\cos 2\theta$
C $\sin 4\theta$ D $\cos 4\theta$

Difference of squares

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= \cos 2\theta \times 1$

$= \cos 2\theta$ ⮚ B

12.19 Find the exact value of $\tan 2\theta$ if $\tan\theta = 0$ and $0 \le \theta \le 90°$

A 4 **B** –2

C 0 **D** 2

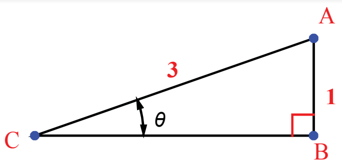
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\ 0}{1-(0)^2} = 0 \Rightarrow C$$

12.20 If $\sin\theta = \frac{-1}{3}$, $270° < \theta < 360°$, find $\sin 2\theta$

A $\frac{-\sqrt{2}}{9}$ **B** $\frac{\sqrt{3}}{9}$

C $\frac{3}{5}$ **D** $\frac{-4\sqrt{2}}{9}$



$$c^2 = a^2 + b^2$$

$$3^2 = a^2 + 1^2$$

$$a = \sqrt{3^2 - 1^2}$$

$$a = \sqrt{8}$$

$$a = 2\sqrt{2} \quad \rightarrow \quad ADJ$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$ positive in II

$$\sin 2\theta = 2\sin\theta\cos\theta$$

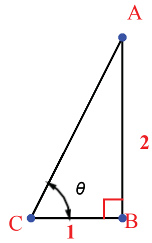
$$= 2 \times \frac{-1}{3}\ \frac{2\sqrt{2}}{3}$$

$$= \frac{-4\sqrt{2}}{9} \Rightarrow D$$

12.21 If $\tan\theta = -2$, $270° < \theta < 360°$, find $\cos 2\theta$

A $\frac{3}{5}$ **B** $\frac{-3}{5}$

C $\frac{-1}{\sqrt{5}}$ **D** $\frac{1}{\sqrt{5}}$



$$c^2 = 1^2 + (-2)^2$$

$$c^2 = 5$$

$$c = \sqrt{5} \quad \rightarrow \quad HYP$$

$$\cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{-2}{\sqrt{5}},$$ θ in quadrant IV

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{+1}{\sqrt{5}}\right)^2 - \left(\frac{-2}{\sqrt{5}}\right)^2$$

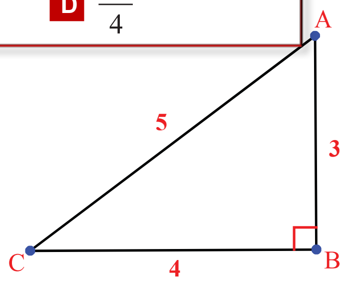
$$= \frac{1}{5} - \frac{4}{5}$$

$$= \frac{-3}{5} \Rightarrow B$$

12.22 If, $\sin\theta = \frac{3}{5}$ and $90° < \theta < 180°$ find $\cot\theta$

A $\frac{4}{3}$ **B** $\frac{-4}{3}$

C $\frac{3}{4}$ **D** $\frac{-3}{4}$



$$\sin\theta = \frac{OPP}{HYB} = \frac{3}{5}$$

Use Pythagorean triple to find ADJ

$$\cot\theta = \frac{ADJ}{OPP}$$

$$= \frac{4}{3}$$

Since θ is in quadrant II

then tan and cot are negative

$$\cot\theta = \frac{-4}{3} \Rightarrow B$$

12.23 If $\cos\theta = \frac{1}{2}$ $0 < \theta < 90°$ then find $\cos\frac{\theta}{2}$

A $\frac{\pm\sqrt{3}}{2}$ B $\frac{-\sqrt{3}}{2}$

C $\frac{3}{4}$ D $\frac{\sqrt{3}}{2}$

$$\cos\frac{\theta}{2} = \pm\frac{\sqrt{1+\cos\theta}}{2}$$
$$= \pm\sqrt{\frac{1+\frac{1}{2}}{2}}$$
$$= \pm\sqrt{\frac{\frac{2}{2}+\frac{1}{2}}{2}}$$
$$= \pm\sqrt{\frac{\frac{3}{2}}{2}}$$
$$= \pm\sqrt{\frac{3}{4}}$$
$$\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$$

Since $\theta < 0 < 90$

$\frac{\theta}{2} < 90$

$\cos\theta$ is positive

D

Analyze the Periodic Functions

Inverse Function	Domain	Range
$\theta = \cos^{-1} x$	$-1 \le x \le 1$	$\theta \le 0 \le \pi$
$\theta = \sin^{-1} x$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$
$\theta = \tan^{-1} x$	All real numbers	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

Note: $\sin^{-1} x$ is the same as are arcsin x

Trigonometric_ ratio [(trigonometric_ $^{-1}$ x]= x ratio)

Example:

Find: $\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\theta$

Let $\theta = \sin^{-1}\frac{1}{2}$

$\sin\theta = \frac{1}{2} \rightarrow \theta = 30$

$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin 30$

$= \frac{1}{2}$

Since $\theta < 0 < 90$

$\frac{\theta}{2} < 90$

$\cos\theta$ is positive

12.24 Find $\cos^{-1}\frac{\sqrt{2}}{2}$

A 180° B 45°

C -45° D 90°

$$\cos^{-1}\frac{\sqrt{2}}{2} = \theta \rightarrow \cos\theta = \frac{\sqrt{2}}{2} \rightarrow \theta = 45°$$

B

12.25 Find $\cos^{-1}(\sin 67)$

A 23° B 67°

C 157° D 33°

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\cos^{-1}(\sin 67) = \cos^{-1}(\cos(90-67))$$
$$= \cos^{-1}(\cos 23)$$
$$= 23°$$

A

12.26 Find x if $\sin^{-1}(\cos x) = \frac{\pi}{6}$

A $\frac{\pi}{3}$ B $\frac{\pi}{4}$

C $\frac{\pi}{6}$ D $\frac{\pi}{2}$

$$\sin(\sin^{-1}(\cos x)) = \sin\frac{\pi}{6}$$
$$\cos x = \sin\frac{\pi}{6}$$
$$\cos x = \frac{1}{2}$$
$$x = \cos^{-1}\frac{1}{2}$$
$$x = \frac{\pi}{3}$$

A

12.27 Find all the values of θ

if $\cos\theta = \frac{1}{2}$ and $0 \le \theta \le 360°$

A 60° B 60° or 120°

C 60° or 300° D 300°

$$\cos\theta = \frac{1}{2} \rightarrow \theta = \cos^{-1}\frac{1}{2}$$
$$= 60°$$

cosθ is positive in quadrants I and IV

$$\rightarrow \theta = 60°, \theta = 360 - 60 = 300°$$

C

12.28 Find the value or values of θ

if $3\cos^2\theta \quad 4\cos\theta - 0$ and $0 \le \theta \le 180°$

A $90°$ **B** $90°$ or $180°$

C $90°$ or $60°$ **D** $60°$

$$3\cos^2\theta - 4\cos\theta = 0$$
$$\cos\theta(3\cos\theta - 4) = 0$$
$$\cos\theta = 0$$
$$\theta = \cos^{-1} 0 = 90$$
or
$$3\cos\theta - 4 = 0$$
$$3\cos\theta = 4$$
$$\cos\theta = \frac{4}{3}$$
$\frac{3}{3}$ is not in the range of cosine function

➢ **A**

12.29 One of the following **is not** a solution for the equation $\sin\theta + \cos\theta\ \tan^2\theta = 0$

A $0°$ **B** $315°$

C $180°$ **D** $90°$

$$\sin\theta + \cos\theta\ \tan^2\theta = 0$$
$$\sin\theta + \cos\theta\ \frac{\sin^2\theta}{\cos^2\theta} = 0$$
$$\sin\theta + \frac{\sin\theta}{\cos\theta}\ \sin\theta = 0$$
$$\sin\theta + \tan\theta\ \sin\theta = 0$$
$$\sin\theta(1 + \tan\theta) = 0$$
$$\sin\theta = 0$$
$$\theta = \sin^{-1} 0$$
$$\theta = 0°, 180°, \text{ or } 360°$$
or
$$1 + \tan\theta = 0$$
$$\tan\theta = -1$$
$$\theta = 135° \text{ or } 315°$$

➢ **D**

12.30 Simplify $\frac{\sec\theta}{\csc\theta}$

A $\sin\theta$ **B** $\cos\theta$

C $\cot\theta$ **D** $\tan\theta$

$$\frac{\sec\theta}{\csc\theta} = \sec\theta \div \csc\theta$$
$$= \frac{1}{\cos\theta} \div \frac{1}{\sin\theta}$$
$$= \frac{1}{\cos\theta} \times \frac{\sin\theta}{1}$$
$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

➢ **D**

12.31 Simplify $\frac{\sin\theta\ \sec\theta}{\cot\theta}$

A $\tan\theta$ **B** $\tan^2\theta$

C $\cot\theta$ **D** $\cot^2\theta$

$$\frac{\sin\theta\ \sec\theta}{\cot\theta} = \frac{\sin\theta\ \frac{1}{\cos\theta}}{\cot\theta}$$
$$= \frac{\tan\theta}{\cot\theta}$$
$$= \tan\theta \times \frac{1}{\cot}$$
$$= \tan\theta \times \tan\theta$$
$$= \tan^2\theta$$

➢ **B**

12.32 Simplify $\frac{\sec\theta}{\sin\theta}(1 \_ \cos^2\theta)$

A $\sec^2\theta$ **B** $\sec\theta$

C $\tan\theta$ **D** $\tan^2\theta$

$$\sec\theta = \frac{1}{\cos\theta},$$
$$\sin\theta + \cos^2\theta = 1 \rightarrow 1 - \cos^2\theta = \sin^2\theta$$
$$\frac{\sec\theta}{\sin\theta}(1 - \cos^2\theta) = \frac{\sec\theta}{\sin\theta}\ \sin^2\theta$$
$$= \sec\theta\ \sin\theta$$
$$= \frac{1}{\cos\theta}\ \sin\theta$$
$$= \tan\theta$$

➢ **C**

12.33 If $\cos x = \sin 42$, then find x

A 90 B 42

C 48 D 138

Recall $\sin 30 = \frac{1}{2}$

$\cos 60 = \frac{1}{2}$

- if $\sin x = \cos y$ then $x + y = 90$

$x + 42 = 90$

$x = 48$

C

12.34 Find $\tan^2\theta - \sec^2\theta$

A 1 B -1

C $\frac{1}{2}$ D $\frac{-1}{2}$

Recall $\tan^2\theta + 1 = \sec^2\theta$

$1 = \sec^2\theta - \tan^2\theta$

$\tan^2\theta - \sec^2\theta = -\left(\sec^2\theta - \tan^2\theta\right)$

$= -1$

B

12.35 Find $\sin 2\theta$ if $\sin\theta + \cos\theta = \frac{5}{3}$

A $\frac{25}{9}$ B $\frac{34}{9}$

C $\frac{16}{9}$ D $\frac{16}{3}$

$(\sin\theta + \cos\theta)^2 = \left(\frac{5}{3}\right)^2$

$\sin^2\theta + 2\sin\cos\theta + \cos^2\theta = \frac{25}{9}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{25}{9}$

$1 + 2\sin\theta\cos\theta = \frac{25}{9}$

$\frac{9}{9} + \sin 2\theta = \frac{25}{9}$ $\quad \sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = \frac{25}{9} - \frac{9}{9} = \frac{16}{9}$

C

CHAPTER (13)

ANALYSING GRAPHS OF FUNCTIONS AND RELATIONS