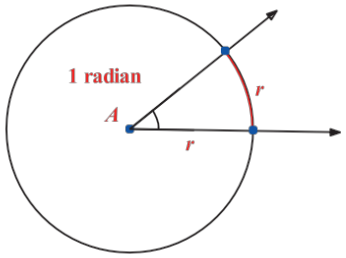
CHAPTER (11)

PERIODIC FUNCTIONS AND TRIGONOMETRY

Radian Measure

A radian is the measure of a central angle that intercepts an arc with length equal to the radius of the circle



- To convert degrees to radians: Multiply by $\frac{\pi}{180^0}$
- To convert radians to degrees:Multiply by $\frac{180^0}{\pi}$
- Example: What is the degree measure of an angle of $\frac{3\pi}{4}$?
- Solution: $\frac{-3\pi}{4} \cdot \frac{180°}{\pi} = -135$
- What is the radian measure of an angle of 27°?

$$27 \cdot \frac{\pi}{180} = \frac{27\pi}{180} \quad \text{Divide by 9}$$
$$= \frac{3\pi}{20}$$

11.1 What is the measure of the minute hand from 12:00 a.m. to 12:00 p.m. in radians?

A 2π B $\frac{3\pi}{2}$

C π D $\frac{\pi}{2}$

The minute hand will move one full circle $(360°)$

$$360° \cdot \frac{\pi}{180} = 2\pi$$

A

11.2 What is the degree measure of an angle $\frac{7\pi}{4}$ radians?

A 345° B 270°

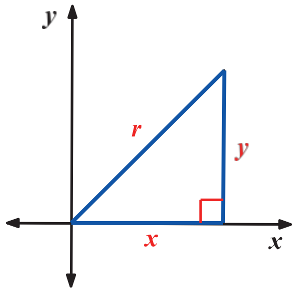
C 330° D 315°

$$\frac{7\pi}{4} \cdot \frac{180}{\pi} = 7 \cdot 45 = 315°$$

D

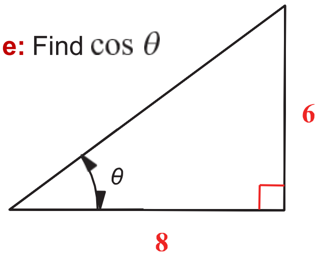
Trigonometric Ratios for a Right Triangle

If θ is an acute angle of a right triangle, x is the length of the adjacent leg (ADJ) y is the length of the opposite leg (OPP), and r is the length of the hypotenuse (HYP) then the trigonometric ratios of θ are as follows:



$\sin\theta = \frac{OPP}{HYP} = \frac{y}{r}$	$\csc\theta = \frac{HYP}{OPP} = \frac{r}{y}$	$\csc\theta = \frac{1}{\sin\theta} \leftrightarrow \sin\theta = \frac{1}{\csc\theta}$
$\cos\theta = \frac{ADJ}{HYP} = \frac{x}{r}$	$\sec\theta = \frac{HYP}{ADJ} = \frac{r}{x}$	$\sec\theta = \frac{1}{\cos\theta} \leftrightarrow \cos\theta = \frac{1}{\sec\theta}$
$\tan\theta = \frac{OPP}{ADJ} = \frac{y}{x}$	$\cot\theta = \frac{ADJ}{OPP} = \frac{x}{y}$	$\tan\theta = \frac{1}{\cot\theta} \leftrightarrow \cot\theta = \frac{1}{\tan\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	
$r = \sqrt{x^2 + y^2}$		

Example: Find $\cos\theta$



Use Pythagorean triples to find the third side $2(3,4,5)$ $\rightarrow (6,8,10)$ $ADJ = 8$ $HYP = 10$

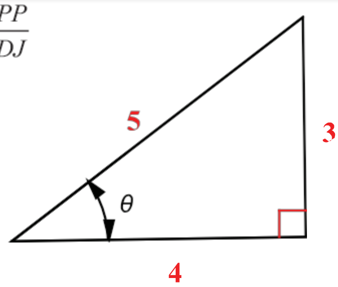
$$\cos\theta = \frac{ADJ}{HYP} = \frac{8}{10} = \frac{4}{5}$$

11.3 Use the triangle ABC to find $\csc\theta - \tan\theta$

A $\frac{1}{2}$ B $\frac{7}{4}$

C $\frac{5}{4}$ D $\frac{3}{4}$

$$\csc\theta - \tan\theta = \frac{HYP}{OPP} - \frac{OPP}{ADJ}$$
$$= \frac{5}{3} - \frac{3}{4}$$
$$= \frac{20}{12} - \frac{9}{12}$$
$$= \frac{21}{12}$$
$$= \frac{7}{4}$$



B

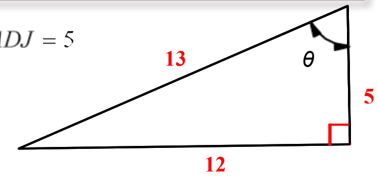
11.4 If $\csc\theta = \frac{13}{12}$ then find $\cos\theta$

A $\frac{12}{13}$ **B** $\frac{5}{12}$

C $\frac{3}{12}$ **D** $\frac{5}{13}$

$\csc\theta = \frac{HYP}{OPP} = \frac{13}{12} \rightarrow ADJ = 5$

$\rightarrow \cos\theta = \frac{ADJ}{HYP} = \frac{5}{13}$



D

θ	0 $0°$	$\frac{\pi}{6}$ $30°$	$\frac{\pi}{4}$ $45°$	$\frac{\pi}{3}$ $60°$	$\frac{\pi}{2}$ $90°$	π $180°$
sin	0				1	0
$\cos\theta$	1				0	-1
$\tan\theta$	0		1	$\sqrt{3}$	Undefined	0

11.5 Give a counter example to $\sin\theta + \cos\theta = 1$ is always true

A $\theta = 360°$ **B** $\theta = 180°$

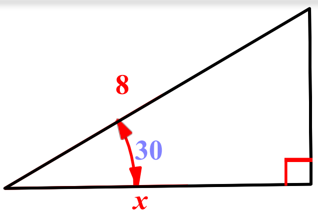
C $\theta = 90°$ **D** $\theta = 270°$

$\sin 180 + \cos 180$

$= 0 + (-1) = -1 \neq 1$

B

11.6 Find x



A 8 **B** 4

C $4\sqrt{3}$ **D** $8\sqrt{3}$

We can use the 30°-60°-90 triangle relationship which is faster but in the following method we applied trigonometry ratios to introduce another way of solving

$HYP = 8,\ ADJ = x$

$\frac{ADJ}{HYP} = \cos\theta = \frac{x}{8}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{x}{8}$

$2x = 8\sqrt{3}$

$x = 4\sqrt{3}$

C

11.7 Find x such that $\cot x$ is undefined

A $x = 135°$ **B** $x = 90°$

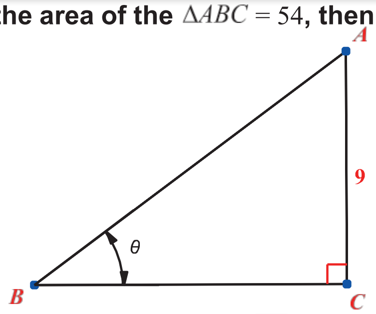
C $x = 60°$ **D** $x = 0°$

$\cot x = \frac{1}{\tan x}$

$= \frac{1}{\tan 0}$

$= \frac{1}{0}$ is undefined

11.8 If the area of the $\Delta ABC = 54$, then find $\tan\theta$



A $\frac{3}{2}$ **B** $\frac{2}{3}$

C $\frac{4}{3}$ **D** $\frac{3}{4}$

$Area = \frac{1}{2}\, b\ h$

$54 = \frac{1}{2}\, b\ \ 9$

$\frac{2}{1} \cdot \frac{1}{9} \cdot 54 = b$

$12 = b$

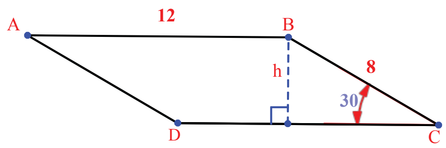
$b = CB = ADJ$

$h = AC = OPP$

$\tan\theta = \frac{OPP}{ADJ} = \frac{9}{12} = \frac{3}{4}$

D

11.9 Find the area of the parallelogram



A 54 **B** 48

C 24 **D** 96

$\sin = \frac{OPP}{HYP} = \frac{h}{8}$

$\sin 30 = \frac{1}{2} = \frac{h}{8}$

$2h = 8$

$h = 4$

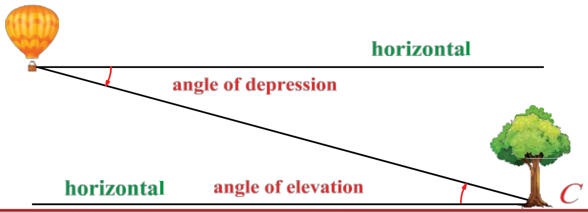
$A = bh$

$= 4\ \ 12 = 48$

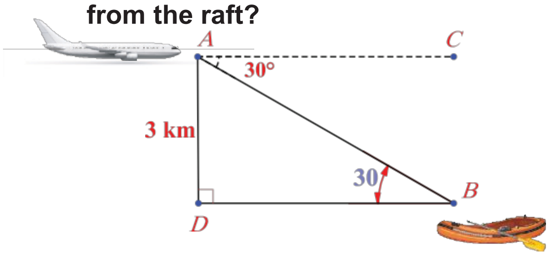
D

Angles of Elevation and Depression

You can use the angles of elevation and depression as the acute angles of right triangles formed by a horizontal distance and vertical height. Hence, trigonometry ratios, 30-60-90, or 45-45-90
can be used.



11.10 **An airplane pilot sights a life raft at 300 angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance d from the raft?**



A $\sqrt{3}$ B 3

C $\frac{\sqrt{3}}{3}$ D $3\sqrt{3}$

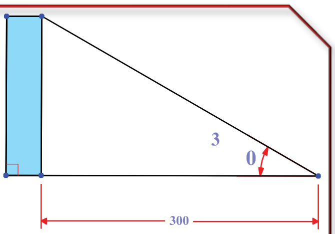
Horizontal distance = longest leg
Shortest leg = 3

$L = S\sqrt{3}$

$= 3\sqrt{3}$

➢ D

11.11 **Find the height of the building if the angle of elevation = 300 and the distance to the base of the building is 300 m.**



A $300\sqrt{3}$ B $30\sqrt{3}$

C $100\sqrt{3}$ D 100

This is a 30-60-90 triangle

height = shortest leg (s)
distance = longest leg

$$L = S\sqrt{3}$$

$$300 = S\sqrt{3}$$

$$S = \frac{300}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{300\sqrt{3}}{3}$$

$$= 100\sqrt{3}$$

Note that you can use $\tan\theta$ also since the height and the Distance represents the OPP and ADJ..

$$\tan\theta = \frac{OPP}{ADJ}$$

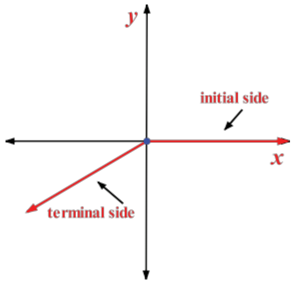
$$\tan 30 = \frac{height}{300}$$

$$\frac{\sqrt{3}}{3} = \frac{h}{300}$$

$$h = \frac{300\sqrt{3}}{3} = 100\sqrt{3}$$

➢ C

Angles in Standard Position

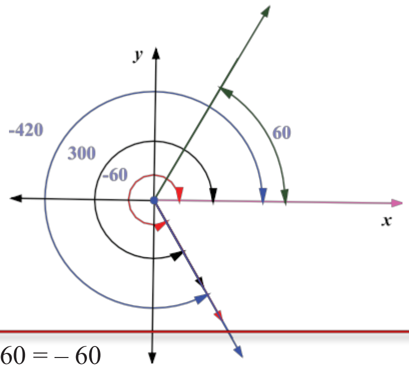


- The measure of an angle is positive when the rotation from the initial side to the terminal side is in the counterclockwise direction.
- The measure is negative when the rotation is clockwise.
- Two angles in standard position are **coterminal angles** if they have the same terminal side.

11.12 **Which of the following angles is not coterminal with any of the other three?**

A 300 B -60

C 60 D -420



$-420 + 360 = -60$
complete one full round
$\rightarrow -420$ *and* -60 are coterminal angles
$-60 + 360 = 300$

$\rightarrow -60$ and $+300$ are coterminal angles
$\rightarrow -60, -420, 300$ are coterminal angles

➢ C

Reference Angle

For any angle in standard position, reference angle is, the smallest, non-negative, angle $0^\circ \le \theta \le 90^\circ$

Quadrant	Measure of angle θ		Measure of reference angle θ´	
I	$0 \to 90^\circ$	$0 \to \frac{\pi}{2}$	$\theta' = \theta$	
II	$90^\circ \to 180^\circ$	$\frac{\pi}{2} \to \pi$	$180^\circ - \theta$	$\pi - \theta$
III	$180^\circ \to 270^\circ$	$\pi \to \frac{3\pi}{2}$	$\theta - 180^\circ$	$\theta - \pi$
IV	$270^\circ \to 360^\circ$	$\frac{3\pi}{2} \to 2\pi$	$360^\circ - \theta$	$2\pi - \theta$

In other words, just think of the angle between terminal side and the nearest *x-axis*.

11.13 Which angle is coterminal to the angle 405^0?

A 90^0　　B 45^0
C 135^0　　D 60^0

$405 - 360 = 45$

➢ B

11.14 If the conterminal side of the angle θ passes through point $(4, -3)$, then find $\sin\theta$

A $\frac{3}{5}$　　B $\frac{4}{5}$
C $\frac{-4}{5}$　　D $\frac{-3}{5}$

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{4^2 + (-3)^2}$$
$$= 5$$

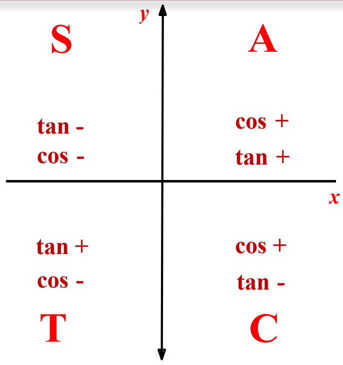
or use pythagrean triple $(3, 4, 5)$

$$\sin\theta = \frac{OPH}{HYP}$$
$$= \frac{y}{r}$$
$$= \frac{-3}{5}$$

➢ D

11.15 Which angle has a negative tan and cos

A 35^0　　B 115^0
C 200^0　　D 305^0



$\cos\theta$ is negative in quadrants II, III

$\tan\theta$ is negative in quadrants II and IV

They are both negative in quadrant II

➢ B

11.16 If $\tan\theta = -\sqrt{3}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then the terminal side of the angle is in quadrant…

A I　　B II
C III　　D IV

$\tan\theta = -\sqrt{3} \to$ negative $\to \theta$ is in quadrant II or IV

$\sin\theta = \frac{\sqrt{3}}{2} \to$ positive $\to \theta$ is in quadrant I or II

θ is in quadrant II

➢ B

11.17 $\sin\theta$ and $\cos\theta$ are both positive in quadrants…

A I　　B II
C III　　D IV

$\sin\theta$ is positive in quadrant I and II
$\cos\theta$ is positive in quadrant I and IV
both are positive in quadrant I

➢ A

11.18 Find $\tan 150^\circ$

A $\frac{\sqrt{3}}{2}$　　B $\frac{-\sqrt{3}}{2}$
C $\frac{1}{2}$　　D $-\frac{1}{2}$

$$\tan 150 = -\tan(180 - 150)$$
$$= -\tan 30$$
$$= \frac{-\sqrt{3}}{2}$$

θ is II
tan is negative

➢ B

11.19 **If** $f(\theta)=\cos\theta$, **the first derivative** $f'(\theta)=-\mathrm{Sin}\,\theta$, **and** $\sin\theta=0.52$ **then find** $\sin(\pi-\theta)$

A 0.48 B 0.26

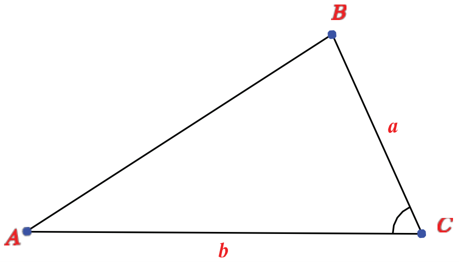
C 0.52 D 1.04

$f'(\theta)$ is an extra information, and we do not need it for this question.

$\mathrm{Sin}(\pi-\theta)=\mathrm{Sin}\,\theta=0.52$

➢ C

Area of a triangle = $\frac{1}{2}$ **a b sin C**

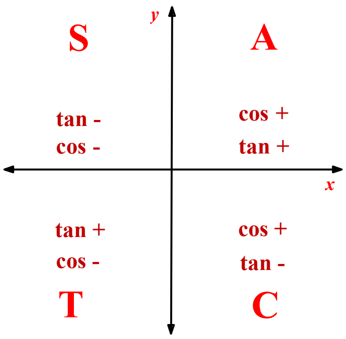


11.20 **The value of** $\frac{\cos\theta}{\tan\theta}$ **is negative when** θ **is in quadrant ...**

A I and II B II and III

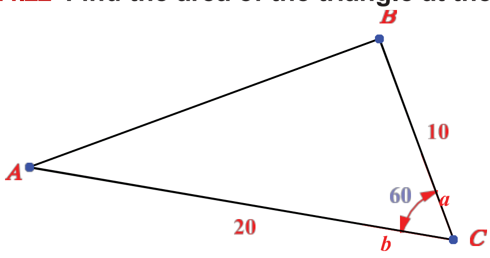
C III and IV D IV and I

The value $\frac{\cos\theta}{\tan\theta}$ will be negative if they have different signs III and IV



➢ C

11.22 **Find the area of the triangle at the right**



A $200\sqrt{3}$ B $100\sqrt{3}$

C $50\sqrt{3}$ D $\frac{\sqrt{3}}{2}$

$$A=\frac{1}{2}ab\sin C$$
$$=\frac{1}{2}\cdot 10\cdot 20\cdot \sin 60$$
$$=\frac{1}{2}\cdot 10\cdot 20\ \frac{\sqrt{3}}{2}$$
$$=50\sqrt{3}$$

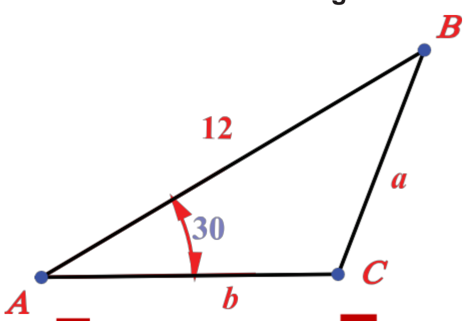
➢ C

Length of an Intercepted arc

For a circle of radius r and a central angle of measure θ, the length of the intercepted arc S is

θ is in radians	θ is in degrees
$S=r\theta$	$S=\frac{\theta}{360}\cdot 2\pi r$

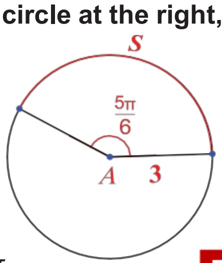
11.23 **If the area of the triangle is 24 then find** $\overline{AC}$



A 8 B 6

C 14 D 10

11.21 **Use the circle at the right, what is the length** S**?**



A $\frac{5\pi}{2}$ B $\frac{2\pi}{5}$

C $\frac{5\pi}{6}$ D $\frac{\pi}{6}$

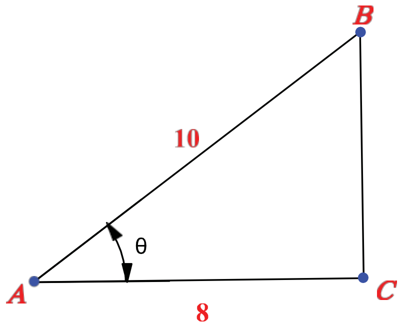
$$S=r\theta$$
$$=3\ .\ \frac{5\pi}{6}$$
$$=\frac{5\pi}{2}$$

➢ A

$$\text{Area}=\frac{1}{2}cb\sin A$$
$$24=\frac{1}{2}\cdot 12\ b\cdot\frac{1}{2}$$
$$24=3b$$
$$b=8$$

➢ A

11.24 **If the area of the triangle is 24, then find** $\csc\theta$



A $\frac{3}{5}$ **B** $\frac{5}{3}$

C $\frac{5}{4}$ **D** $\frac{4}{5}$

$$\text{Area} = \frac{1}{2}bc\sin A$$

$$24 = \frac{1}{2}\,8\cdot 10\sin\theta$$

$$24 = 40\cdot \operatorname{Sin}\theta$$

$$\sin\theta = \frac{24}{40}$$

$$= \frac{3}{5}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$= \frac{5}{3}$$

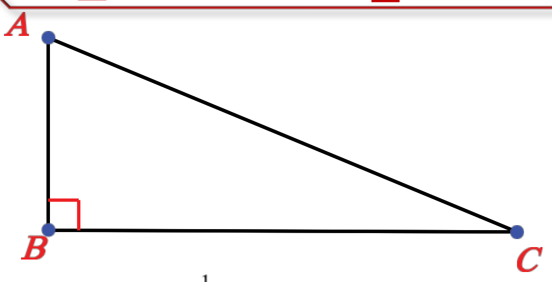
➤ B

11.25 **Find** x **if the area of the triangle** ABC **is** 120

$$AB = \frac{2x+4}{x-9} \quad CB = \frac{24x}{2x+4}$$

A 5 **B** 6

C 10 **D** 90



$$A = \frac{1}{2}ac\sin B$$

$$= \frac{1}{2}\cdot\frac{24x}{2x+4}\cdot\frac{2x+4}{x-9}\cdot\sin 90$$

$$120 = \frac{1}{2}\cdot\frac{24x}{(x-9)}\cdot 1$$

$$120 = \frac{12x}{x-9}$$

$$120(x-9) = 12x$$

$$10(x-9) = x$$

$$10x - 90 = x$$

$$9x = 90$$

$$x = 10$$

➤ C