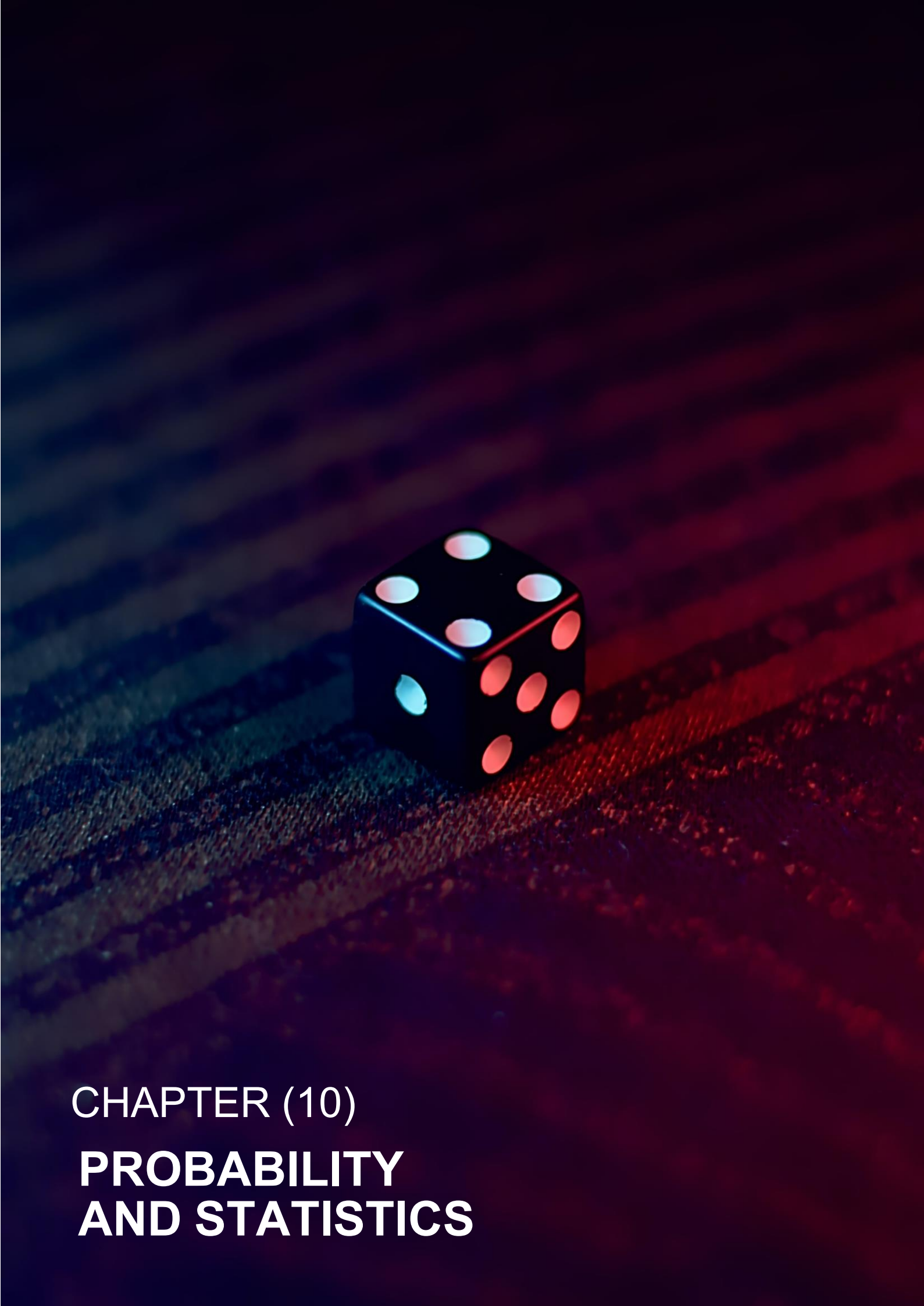


CHAPTER (10)

PROBABILITY AND STATISTICS

* **Probability** measures how likely it is for an event to occur

* Probability for any even X, $P(X)$

- The probability of an impossible event is 0 (or 0%)
- The probability of a certain event is 1 (or 100%)
- Otherwise the probability of an event is a number between 0 and 1 (or between 0% and 100%)

$$0 \le P(X) \le 1$$

Experimental probability can be obtained by gathering data from observation. Each observation is an experiment or a trial

$$P(\text{event}) = \frac{\text{Number of times the event occurs}}{\text{Number of trials}}$$

Counting Principle

If an event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by even N can occur in $m \cdot n$ ways

10.1 How may way can you choose clothes from the closet

Skirt	2	blouse	5	pants	4

A 20 **B** 40

C 10 **D** 11

Using the fundamental counting principle

$2 \times 5 \times 4 = 40$

➢ **B**

10.2 How many ways can you arrange 4 books on 3 shelves

A 9 **B** 4

C 2 **D** 81

Each book can be arranged in three ways, but we have 4 books so it is $3 \times 3 \times 3 \times 3 = 81$

➢ **D**

10.3 How many elements in a sample space (outcomes) of drawing two cards with replacement from a deck of cards numbered from 1 to 10

A 50 **B** 100

C 90 **D** 16

The number of first possible outcomes are 10, since the card will be replaced again, then the number of the second outcomes is 10 too, using counting principle

$10 \times 10 = 100$

➢ **B**

10.4 What is the theoretical probability of getting an even number on one roll of a standard number cube?

A $\frac{1}{2}$ **B** $\frac{2}{3}$

C $\frac{1}{4}$ **D** $\frac{1}{6}$

There are six equally likely outcomes, 1,2,3,4,5 and 6

An even number occurs 3 times (2,4,6)

$\frac{3}{6} = \frac{1}{2}$

➢ **A**

10.5 What is the theoretical probability of getting a sum of 5 on one roll of two standard number cubes

Note: It will be the same question if it is: two rolls of a standard number cube.

A $\frac{2}{3}$ **B** $\frac{5}{36}$

C $\frac{1}{9}$ **D** $\frac{1}{6}$

There are 36 possible equally likely out comes. The favorable outcomes are those with a sum of 5

$(1,4),\ (2,3),\ (4,1),\ (3,2)$

$P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9}$

➢ **C**

Using **factorial notation** you can write 3.2.1 as 3! read three factorial

n factorial is $n! = n \times (n-1)!$

$0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note: $5! = 5 \times 4! = 5 \times 24 = 120$

Permutation is an arrangement of terms in a particular order. In other words, the order of the terms is important

The number of permutations of n items of a set arranged r items at a time is

$${}_n P_r = \frac{n!}{(n-r)!} \quad \text{for } 0 \le r \le n$$

Find permutation mentally: Find ${}_5 P_3$

Start from 5 and multiply by $5 \cdot (5\text{-}1) \cdot (5\text{-}2)$ (r times)

$5 \cdot 4 \cdot 3 = 60$

10.6 If $n!\quad 720$, then find $(n-1)!$

A 720 B 120

C 719 D 360

$n! = 720$

$6! = 720 \rightarrow$

$n = 6$

$n-1 = 5$

$5! = 120$

➢ B

10.7 If ${}_nP_2 = 42$, then find n^2

A 7 B 49

C 6 D 36

$${}_n\mathrm{P}_r = \frac{n!}{(n-r)!}$$

$${}_n\mathrm{P}_2 = \frac{n!}{(n-2)!}$$

$$42 = \frac{n(n-1).(n-2)}{(n-2)!}$$

$42 = n(n-1)$

$42 = 7 \cdot 6$

$\rightarrow n = 7,\ n^2 = 49$

➢ B

10.8 In how many ways can 10 runners finish, first, second and third?

A 20 B 720

C 120 D 40

The order is important then if it should be permutation

${}_{10}\mathrm{P}_3 \quad 10 \times 9 \times 8 = 720$

➢ B

Repeated Permutation

Number of different permutation of n objects where there are r_1 repeated items, r_2 repeated items r_k, repeated Items is $\dfrac{n!}{r_1!r_2!...r_k!}$

If n objects are arranged in a circle **without a reference point** then there are $(n-1)!$ permutations

If n objects are arranged in a circle **with a fixed Reference point,** then there are $n!$ permutations.

10.9 what is the probability to get the word. PARALLEL from the letters: L E L R A P A L

A $\dfrac{1}{1680}$ B 1680

C 3360 D $\dfrac{1}{3360}$

The number of letters $n = 8$

r_1 L is repeated 3 times

r_2 A is repeated 2 times

$$\frac{n!}{r_1!r_2!} = \frac{n!}{r_1!r_2!}$$

$$= \frac{8!}{3!2!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!2!} = 3360$$

$$\rightarrow P(\text{parallel}) = \frac{1}{3360}$$

➢ D

10.10 In how many ways can 7 persons sit round a table?

A 120 B 5040

C 720 D 360

They can sit in any order and there is not any fixed reference point

$(n-1)! = (7-1)!$

$6! = 720$

➢ C

10.11 In how many ways can 6 students sit round a table if Ahmad sat near the window?

A 720 B 120

C $\dfrac{1}{120}$ D $\dfrac{1}{720}$

Ahmad sat near the window → reference point

$n = 6$

$n! = 6! = 720$

➢ A

Combinations is a selection in which order does not matter

The number of combination of n items of a set chosen r items at a times is

$${}_nC_r = \frac{n!}{r!(n-r)!} \quad \text{for } 0 \le r \le n$$

Find combination mentally: Find ${}_7C_3$

$$\frac{\text{Start from 7 descending } r \text{ times}}{\text{start from 1 to } r}$$

$$= \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} = 7 \cdot 5 = 35$$

10.12 If two number were chosen from 1 to 20, then what is the probability that the two numbers are 7 and 12

A 140 **B** $\frac{1}{190}$

C 380 **D** $\frac{1}{380}$

Since the order is not important combination is used

$${}_{20}C_2 = \frac{20 \cdot 19}{1 \cdot 2} = 190$$

$$P\ (7,12) = \frac{1}{190}$$

➢ **B**

10.13 A bag contains 3 red marbles and 4 blue marbles. If two marbles were chosen randomly, then find the probability of getting two different marbles

A $\frac{7}{12}$ **B** $\frac{1}{12}$

C $\frac{4}{7}$ **D** $\frac{2}{7}$

Choosing two different marbles out of 7 marbles and the order does not matter

→ combination ${}_7C_2$

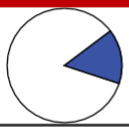
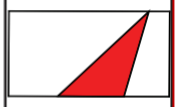
$${}_7C_2 = \frac{7 \cdot 6}{1 \cdot 2} = 21$$

Use counting principle to choose different marbles $3 \times 4 = 12$

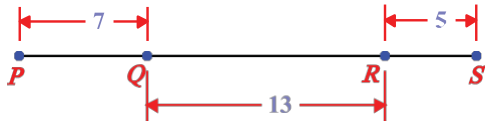
P (different marbles) = $\frac{12}{21} = \frac{4}{7}$

➢ **C**

Geometric probability

Model	Length	Angle measure	Area
Example			
Event	All points on $\overline{BC}$	All points in the shaded region	All points in the triangle
Sample Space	All points on	All points in the circle	All points in the rectangle
Probability	$P = \frac{BC}{AD}$	$P = \frac{\text{Measure of angle}}{360°}$	$P = \frac{\text{Area of triangle}}{\text{Area of rectangle}}$

10.14 A point is chosen randomly on PS. Find the probability that the point is on RS



A $\frac{1}{5}$ **B** $\frac{5}{7}$

C $\frac{1}{4}$ **D** $\frac{5}{13}$

$$P(\text{point on } \overline{RS}) = \frac{RS}{PS} = \frac{5}{25} = \frac{1}{5}$$

➢ **A**

10.15 Find the probability that the spinner lands on the blue sector

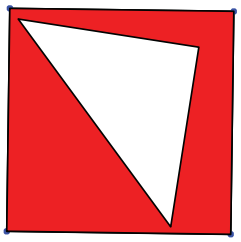
A $\frac{1}{3}$ **B** $\frac{1}{9}$

C $\frac{1}{4}$ **D** $\frac{1}{8}$

$$P(\text{spinner on blue}) = \frac{40}{360} = \frac{1}{9}$$

➢ **B**

10.16 If the area of the square is 24 and the area of the triangle is 8, then find the probability that a randomly chosen point is in the shaded area



A $\frac{3}{4}$ **B** $\frac{1}{4}$

C $\frac{1}{3}$ **D** $\frac{2}{3}$

Area of the shaded part = 24 - 8 = 16

$$P(\text{point in shaded}) = \frac{16}{24} = \frac{2}{3}$$

➢ **D**

Dependent and Independent Events

Probability of A and B (**Independent**)
If A and B are independent events,

$$P(A \text{ and } B) \rightarrow P(A \cap B) = P(A) \cdot P(B)$$

Example of independent events

- Pick a card with replacement then picking another card
- Roll a number cube, then spin a spinner

Probability of A and B (Dependent)
If A and B are dependent events

$$P(A \text{ and } B) \rightarrow P(A \cap B) = P(A) \cdot P(B \setminus A)$$

Example of dependent events

- Pick one flash card, then another from a stack of 30 flash cards
- A month is selected at random then a day of that month of that month is elected.

Conditional Probability

- The probability that an event B will occur given that another event has already occurred is called **conditional probability.**
- You write conditional probability of event B given that event A occurs as $P(B \setminus A)$ and read: The probability of event B given even A
- For any two events A and B with $P(A) \neq 0$

$$(B \setminus A) = \frac{P(A \cap B)}{P(A)}$$

10.17 **If a standard number cube rolled 8 times and all outcomes were odd numbers then what is theoretical probability to get an even number in the 9th roll?**

A $\frac{1}{18}$ **B** $\frac{1}{9}$

C $\frac{1}{3}$ **D** $\frac{1}{2}$

Rolling a cube 8 time one time after the other are independent events.

$$\frac{\text{Get even number } 9^{\text{th}} \text{ roll}}{\text{all out comes}} = \frac{3}{6} = \frac{1}{2}$$

➢ D

10.18 **A Box Contains 4 blue marbles and 6 red marbles. If two marbles were drawn randomly without replacement. What is the probability to get a blue marble in the second time if the first marble was red?**

A $\frac{2}{5}$ **B** $\frac{2}{3}$

C $\frac{1}{3}$ **D** $\frac{3}{5}$

The Total number of marbles: $4+6=10$
After drawing the first marble the total number of marble decreased by one but the blue marbles are the same.

$$n = 10 - 1 = 9$$

$$P(\text{Blue}) = \frac{6}{9} = \frac{2}{3}$$

➢ B

10.19 **What is the probability to get 3 heads when tossing a coin 3 times?**

A $\frac{1}{6}$ **B** $\frac{1}{8}$

C $\frac{1}{12}$ **D** $\frac{1}{4}$

Tossing a coin 3 times are independent events.

$$P(\text{Head 1 toss}) = \frac{1}{2}$$

$$P(\text{Head 3 times tossing}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

➢ B

10.20 **What is the probability of getting a number greater than 3 and a head on one roll of a standard number cube and on tossing a coin.**

A $\frac{1}{6}$ **B** $\frac{1}{4}$

C $\frac{1}{2}$ **D** $\frac{2}{3}$

Let A Represents getting number greater than 3: {4,5,6} and B represents getting a head: {head}

$$P(A) \frac{3}{6} \quad \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

➢ B

10.21 **What is the Probability of getting 5 on one roll of two number cubes at the top of one cube if the sum of the two numbers was 8.**

A $\frac{5}{36}$ **B** $\frac{3}{5}$

C $\frac{1}{2}$ **D** $\frac{2}{5}$

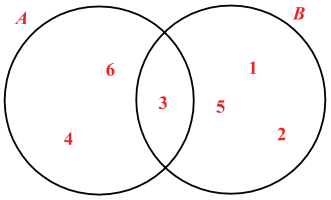
The sample space is

$$\{(3,5),\ (4,4),\ (2,6),\ (6,2),\ (5,3)\}$$

$$P(\text{Getting 5 on one cube}) = \frac{\text{number of times 5 occurs}}{\text{sampl space}} = \frac{2}{5}$$

➢ D

10.22 **The Venn Diagram Represents rolling a number Cube. Find** $P(A \setminus B)$



A $\frac{1}{4}$ **B** $\frac{1}{3}$

C $\frac{1}{2}$ **D** $\frac{1}{6}$

Total Number of outputs is 6.

$$P(B) = \frac{4}{6} = \frac{2}{3}$$

$$P(A \cap B) = \frac{1}{6}$$

$$P(A \setminus B) = \frac{p(A \cap B)}{p(B)} = \frac{\frac{1}{6}}{\frac{2}{3}}$$

$$= \frac{1}{6} \times \frac{3}{2}$$

$$= \frac{1}{4}$$

➢ A

Conditional Probability and tables

$P(A \text{ given that } C \text{ occurs})$

$\rightarrow P(A/C) = \frac{w}{x+z}$

$P(D \text{ Given that } B \text{ occurs})$

$\rightarrow P(D/B) = \frac{z}{x+z}$

	A	*B*
C	*w*	*x*
D	*y*	*z*

10.23 **What is the probability that a customer pays the bill online given that the customer is male.**

	Online	Cash on delivery
Male	12	8
Female	24	6

A $\frac{3}{5}$ **B** $\frac{2}{5}$

C $\frac{3}{4}$ **D** $\frac{5}{6}$

$$P(\text{male}) = 12 + 8 = 20$$

$$P(\text{online} \cap \text{male}) = 12$$

$$P(\text{online} / \text{male}) = \frac{P(\text{online} \cap \text{male})}{P(\text{male})}$$

$$= \frac{12}{20} = \frac{3}{5}$$

➢ A

10.24 **What is the probability that a person is a female given that the person applies shampoo directly to the head.**

	Directly to the head	Into hand First
Male	2	18
Female	6	24

A $\frac{1}{5}$ **B** $\frac{1}{4}$

C $\frac{6}{6}$ **D** $\frac{3}{4}$

$$P(\text{directly}) = 2 + 6 = 8$$

$$P(\text{female} \cap \text{directly}) = 6$$

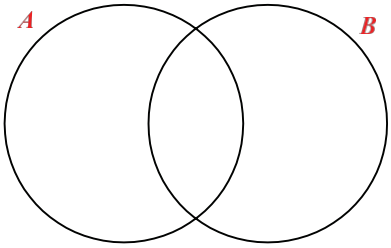
$$P(\text{female} / \text{directly}) = \frac{P(\text{female} \cap \text{directly})}{P(\text{directly})}$$

$$= \frac{6}{8} = \frac{3}{4}$$

➢ D

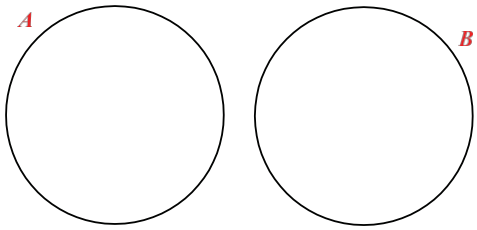
Probability of A or B

Two events that have common out comes are called **exclusive** events.



$$P(A \text{ or } B) \rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Two events that cannot happen at the same time are **mutually exclusive** events.



$$P(A \cap B) = 0$$

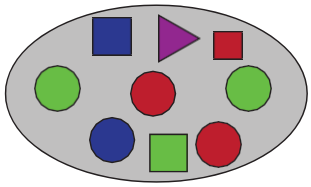
$$P(A \text{ or } B) \rightarrow P(A \cup B) = P(A) + P(B)$$

Example

Roll a standard number cube:

- Rolling a 2 and 3 → **mutually** exclusive
- Rolling an even number and a multiple of 3 → Not **mutually** exclusive

10.25 If you select a shape at random. What is the probability that the shape round or green?



A $\frac{2}{9}$ B $\frac{3}{9}$

C $\frac{6}{9}$ D $\frac{8}{9}$

P (round or green) =
P (round) + P (green) – P (round and green)

$$= \frac{5}{9} + \frac{3}{9} - \frac{2}{9}$$

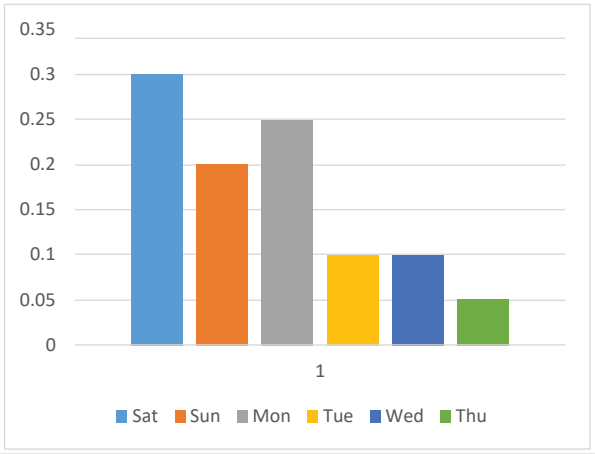
$$= \frac{6}{9}$$

➢C

10.26 What is the probability that a day is chosen at. random to be Sun on Tue.

A 0.3 B 0.5

C 0.7 D 0.8



$P(\text{Sun}) = 0.2$
$P(\text{Tue}) = 0.1$
$P(\text{Sun} \cap \text{Tue}) = 0$
$P(\text{Tue or Sun}) = P(\text{Sun}) + P(\text{Tue})$
$= 0.2 + 0.1 = 0.3$

➢A

10.27 What is the probability of getting two odd numbers or their sum is 4 on one roll of two standard number cubes?

A $\frac{10}{36}$ B $\frac{9}{36}$

C $\frac{12}{36}$ D $\frac{2}{36}$

Two odd numbers
$= \{(1,3),(1,5),(3,5),(3,1),(5,1),(5,3),(1,1),(3,3),(5,5)\}$
The sum is 4 $= \{(1,3),\ (3,1),\ (2,2)\}$

Odd and Sum is 4 $\rightarrow \{(1,3),\ (3,1)\}$

$P(\text{odd}) = \frac{9}{36}$

$P(\text{sum } 4)\ \frac{3}{36}$

$P(\text{odd} \cap \text{Sum is } 4) = \frac{2}{36}$

$P(\text{odd} \cup \text{Sum is } 4) = P(\text{odd}) + P(\text{Sum is } 4) - P(\text{odd} \cap \text{Sum is } 4)$

$$= \frac{9}{36} + \frac{3}{36} - \frac{2}{36}$$

$$= \frac{10}{36}$$

➢A

Complementary Events

Two events are said to be complementary when one event occurs if and only if the other does not.

- Probabilities of two complementary events add up to 1
- Let A and A' be to complementary events,

$$P(A') = 1 - P(A)$$
$$P(A) = 1 - P(A')$$

10.28 **If the probability of the students who passed the test is 85% then, what is the probability of who failed?**

A 85% **B** 15%

C 50% **D** 25%

since the two events are complementary

P(failed) = 1 – P(passed)
= 1 – 85% = 15% ➢ **B**

Samples and Surveys

- A population is all members of a set
- A sample is part of population.
- You can get statistical information about population by studying a sample of the population

1- **Observational study**: To observe members of a sample in such a way they are not affected by the study.

2- **Controlled Experiment:** To divide a sample into two groups. Then impose a treatment on one group but not on the other "control" group. Then to compare the effect on the treated group to the control group.

3- **Survey:** To ask every member of the sample a set of questions

Sampling Types and methods

1- **Convenience Sample:** Select any member of the population who are conveniently and readily available

2- **Self-Selected Sample:** Select only members of the population who volunteer for the sample

3- **Systematic Sample:** Order the population in some way, and the select from it at regular intervals

4- **Random Sample:** all members of the population are equally likely to be chosen.

Margin of error $= \pm\frac{1}{\sqrt{n}}$ n = number of population

10.29 **A random of 400 students divided randomly into two groups. The first group was allowed to use calculator but the other group was not allowed to. What type of study is this?**

A Observation **B** Controlled experiment

C Survey **D** Random Study

In a controlled experiment the sample is divided into two groups

➢ **B**

10.30 **Tracing the effect of smoking on 100 volunteers through 10 years What type of study is this?**

A Observation Study **B** Survey

C Controlled experiment **D** Random Study

This study is done by observing the volunteers

➢ **A**

10.31 **On a survey, 85% of 400 students said that they like math.**
Find the margin of error for this study.

A 10.5 **B** ± 0.05

C ± 5 **D** ± 0.005

$$\text{Margin of error} = \pm\frac{1}{\sqrt{400}}$$
$$= \pm\frac{1}{20} = \pm 0.05$$

➢ **B**

10.32 **On a survey, 75% of 625 students study more than 3 hours a day.**
What is the range of their present?

A 71% to 79% **B** 70% to 80%

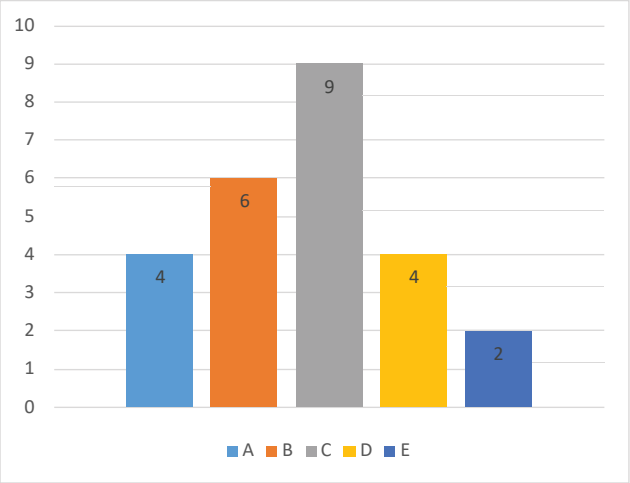
C 72% to 18% **D** 74% to 76%

Step 1: margin of error

$$= \pm\frac{1}{\sqrt{625}} = \pm\frac{1}{25} = \pm 0.04 = \pm 4\%$$

Step 2: 75% + 4% = 79%
75% - 4% = 71% ➢ **A**

10.33 **Use the histogram chart to find the percent of students who score maximum B in Algebra test.**



A 60% **B** 84%

C 24% **D** 42%

Total number of students
$= 2+4+9+6+4 = 25$
Maximum got B means $= B+C+D+E$
$= 6+9+4+2$
$= 21$

Percentage $= \frac{21}{25} = 84\%$ ➢ **B**

Measures of central Tendency

Mean	sum of data values / Number of data values	Used if the data values have no outlier
Median	1) arrange the data in order 2) Choose the middle value of odd number data values 3) Choose the mean of two middle values for an even number of data values	Used if the data values have outlier and no gabs
Mode	The most frequently occurring values	Used if the data value have repeated values

10.34 **One of the following is not of the central tendency measures**

A Mean **B** Median

C Mode **D** Standard Deviation

Measures of central tendency are 3: mean, median and mode ➢ **D**

10.35 **Determine the best central tendency measures to be used for the following data**

A Mean **B** Median

C Mode **D** Standard Deviation

3	3	7	5	3	4	5
5	4	3	3	4	2	1
4	3	4	8	5	5	4

3, 4, 5 occurs frequently ➢ **C**

10.36 **Find the median of the following set of data 120, 62, 56, 81, 93, 69**

A 75 **B** 77

C 68.5 **D** 81

Rewrite the data in order 56, 63, 69, 81, 93, 120

Median $= \frac{69+81}{2} = \frac{150}{2} = 75$ ➢ **A**

Standard Deviation

Variance and standard deviation are measures showing how much data values deviate from the mean

Symbol	Read	Represents
σ	Sigma	Standard deviation
σ^2	Sigma squared	Variance
$\bar{x}$ or μ	x bar or meo	Mean of the n values in a data set

Steps to find variance and standard deviation

1) Find the mean $\bar{x}$ of the n values in the data set

2) Find the difference $x-\bar{x}$ between each value x and mean

3) Square each difference $(x-\bar{x})^2$

4) Find the mean of these squares = variance

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n}$$

$P(A \text{ or } B) \rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5) The square root of the variance is the standard deviation

$$\sigma = \sqrt{\frac{(x-\bar{x})^2}{n}}$$

- Range is the deference between the greatest and
- least value |Greatest – Least |
- Standard deviation varies directly with the range

10.37 **Which data set has the greatest standard deviation**

A 24,13,25,16,8 **B** 25,12,22,16,12

C 30,14,18,12,10 **D** 30,12,24,27,16

Find the range of each data set option C has the greatest range

$|30 - 10| = 20$

10.38 **If the standard deviation of a data set is 9 then its variance is**

A 3 **B** 81

C 9 **D** 27

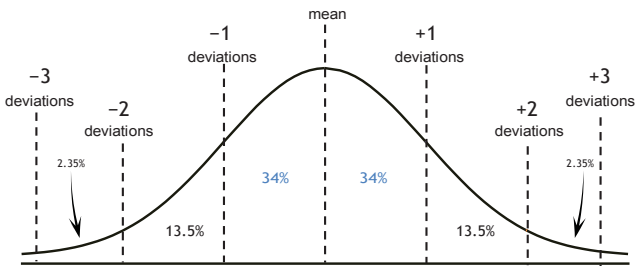
$\text{variance} = \sigma^2 = (\text{standard deviation})^2$

$= 9^2 = 81$

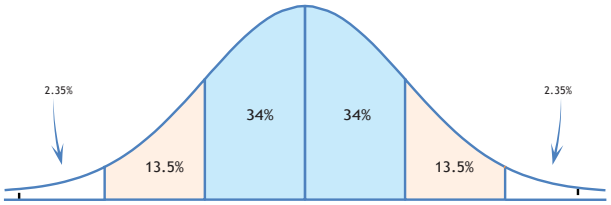
➢ **B**

Normal Distribution

A normal distribution has data that vary randomly from the mean. The graph of a normal distribution is a normal curve.



10.39 **The area under the normal curve is …**

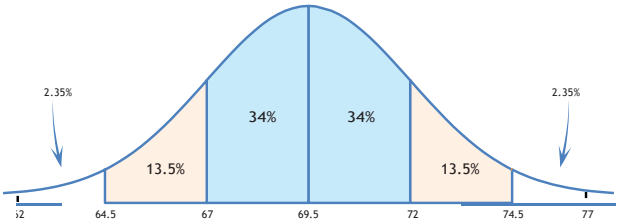


A $\frac{1}{2}$ **B** $\frac{1}{4}$

C $\frac{3}{4}$ **D** 1

➢ **D**

10.40 **What percent of adult male between 67 in and 74.5 in tall**



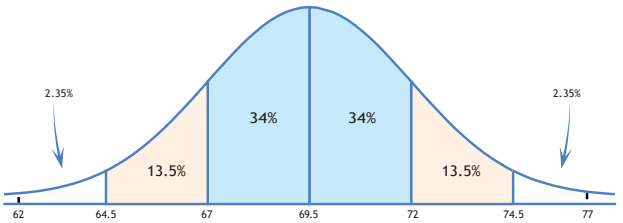
A 81.5% **B** 68%

C 47.5% **D** 40%

$P(67 < \text{height} < 74.5) = 0.34 + 0.34 + 0.135 = 0.815$

➢ **A**

10.41 **In a group of 2000 adult males, what is the probability would you expect to be taller than 72 in?**



A 0.68 **B** 47.5

C 0.16 **D** 0.5

$P(\text{height} > 72) = 0.5 - 0.34 = 0.16$

➢ **C**