CHAPTER (7)

TRANSFORMATION

- In a transformation the original figure is the **preimage**. The resulting figure is the **image**.
- An **isometry** is a transformation in which the preimage and image are congruent.
- A transformation of geometric figure is a change in the position (**translation**), flipping (**reflection**), turning (**rotation**), or size (**dilation**).

(1) Translation

(1) Translation is transformation that maps all points of a figure the same distance in the same direction

$$P(x,y) \longrightarrow P'(x+h,\ y+v)$$

where h is the horizontal shift and v is vertical shift

Example

What is the image of the point $P(2,1)$ for the translation?

$$(x,y) \longrightarrow (x-2,\ y-5)$$

$$P\ (2,1) \longrightarrow P'(2-2,\ 1-5) \longrightarrow P'\ (0,-4)$$

1. Find the image of the point $(6,2)$ for the translation three units to right and four units down

A $(9,-2)$ **B** $(-2,9)$

C $(3,6)$ **D** $(6,3)$

Three units to the right $\longrightarrow x+3$

Four units down $\longrightarrow y-4$

$$(x,y) \longrightarrow (x+3,\ y-4)$$

$$(6+3,\ 2-4)$$

$$(9,-2)$$

➢ **A**

2. What is a rule that describes the translation? $P(-3,4)$ to $P'(5,2)$

A $(x-8,\ y+2)$ **B** $(x+2,\ y-8)$

C $(x-2,\ y+8)$ **D** $(x+8,\ y-2)$

Horizontal Change: $-3+h=5$

$$h=+8$$

$$x'=x+h$$
$$x'=x+8$$

Vertical change: $4+v=2$

$$v=-2$$

$$y'=y+v$$

$$y'=y-2$$

$$P(x,y) \longrightarrow P'(x+8,\ y-2)$$

➢ **D**

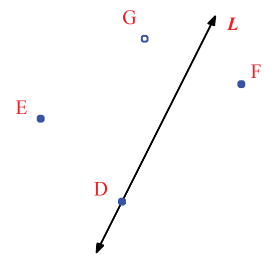
(2) Reflection

If a point A is on the line r, then the image of A is itself $(A'=A)$

If a point B is not on line r, then r is the perpendicular bisector of BB'

Point	Axis of reflection	image
(a,b)	$x-$axis	$(a,-b)$
(a,b)	$y=x$	(b,a)
(a,b)	$y-$axis	$(-a,b)$

3. Find the image of D across the line L



A E **B** D

C F **D** G

The point D is on the of reflection line then the image of D is itself $D=D'$ ➢ **B**

4. Find the image of the point $(2,-3)$ across x-axis

A $(2,-3)$ **B** $(-2,-3)$

C $(-2,3)$ **D** $(2,3)$

$$(a,b) \xrightarrow{\text{across } x\text{-axis}} (a,-b)$$

$$(2,-3) \longrightarrow (2,+3)$$

➢ **D**

5. Find the image of the point $(6,3)$ across y-axis

A $(6,3)$ **B** $(-6,3)$

C $(6,-3)$ **D** $(3,6)$

$$(a,b) \xrightarrow{\text{across } y\text{-axis}} (-a,b)$$

$$(6,3) \longrightarrow (-6,3)$$

➢ **B**

6. What is the image of the point $(3,-1)$ across the line $y=x$

A $(-3,1)$ **B** $(1,-3)$

C $(-1,3)$ **D** $(-1,3)$

Reflection $(a,b) \xrightarrow{\text{across } y=x} (b,a)$

$(3,-1) \longrightarrow (-1,3)$ ➤ **D**

7. If $A'(4,7)$ is the image of $A(7,4)$ then determine the line or point of reflection.

A The origin **B** $x-\text{axis}$

C $y-\text{axis}$ **D** The line $y = x$

$A(7,4) \longrightarrow A'(4,7)$

$(a\ b) \longrightarrow (b,a)$

Across $y = x$ ➤ **D**

(3) Rotation

The rotation is applied counterclockwise unless it was mentioned clockwise

The rotation about the origin

Point A	Angle of rotation	Image A'
(a,b)	90^o	$(-b,a)$
(a,b)	180^o	$(-a,-b)$
(a,b)	270^o	$(b,-a)$
(a,b)	$360°$	(a,b)

8. What is the image of the point $(5,3)$ for a $90°$ rotation about the origin

A $(5,3)$ **B** $(-3,5)$

C $(-3,-5)$ **D** $(5,-3)$

$(a,b) \xrightarrow{\text{Rotation}90°} (-b,a)$

$(5,3) \longrightarrow (-3,5)$ ➤ **B**

9. What is the image of the point for $(4,-2)$ $180°$rotation about the origin

A $(4,-2)$ **B** $(-4,2)$

C $(+4,-2)$ **D** $(-4,-2)$

$(a,b) \xrightarrow{\text{Rotation}180°} (-a,-b)$

$(4\ -2) \longrightarrow (-4,\ -\ (-2))$

$(-4,2)$ ➤ **B**

10. What is the image of the point $(4,0)$ for $270°$ rotation

A $(0,4)$ **B** $(4,0)$

C $(0,-4)$ **D** $(0,4)$

$(a,b) \xrightarrow{\text{Rotation}270^\circ} (b,-a)$

$(4,0) \longrightarrow (0,-4)$ ➤ **C**

11. What is the image of the point $(-5,1)$ for $360°$rotation

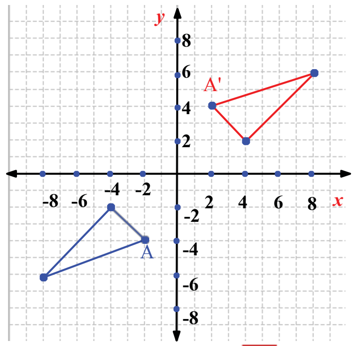
A $(-5,1)$ **B** $(5,-1)$

C $(1,-5)$ **D** $(-1,5)$

$(a,b) \xrightarrow{\text{Rotation}360°} (a,b)$

$(-5,1) \longrightarrow (-5,1)$ ➤ **A**

12. What is the angle of rotation about the origin for the ΔABC to $\Delta A'B'C'$



A $90°$ **B** $180°$

C $270°$ **D** $360°$

$(-2,-4) \longrightarrow (2,4)$

$(a,b) \xrightarrow{\text{Rotation}180°} (-a,-b)$ ➤ **B**

Rotation of Regular Polygons

You can find of the rotation of a regular polygon (that has n sides) about its center by $\dfrac{360}{n}$

Example:

Find the angle of rotation of a hexagon about its center

Solution:

hexagon has 6 sides

$$\text{Angle of rotation} = \frac{360}{n} = 60°$$

13. Find the angle of rotation of a regular pentagon

A $50°$ **B** $72°$

C $120°$ **D** $60°$

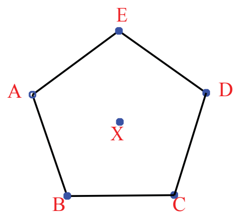
$n = 5$

$$\text{Angle of rotation} = \frac{360}{n}$$

$$= \frac{360}{5}$$

$$= 72°$$ **B**

14. What is the image of $216°$ rotation of E about X



A A **B** B

C C **D** D

The pentagon has 5 sides $\longrightarrow n = 5$

$$\text{Angle of rotation} = \frac{360}{5} = 72°$$

$$= 216 \div 72 = 3$$

So, a $216°$ rotation about X moves each vertex three vertices counterclockwise

$$E \xrightarrow{\text{3 vertices}} C$$ **C**

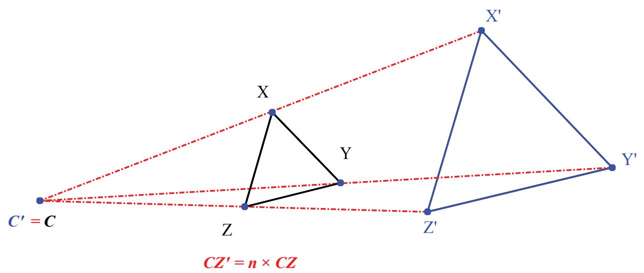
(4) Dilations

A dilation with center C and scale factor n:

- The image of C is itself $(C' = C)$
- For any other point R,

  $\longrightarrow R'$ is on $\overrightarrow{CR}$

  $\longrightarrow CR' = n \cdot CR$, or $n = \frac{CR'}{CR}$



- The image of the dilation is similar to the preimage, not congruent

Scale factor n:	$n > 1$	$0 < n < 1$	$n = 1$
Dilation is	Enlargement	Reduction	Congruent (no dilation)

- The dilation image of a point $P(x, y)$ is found by multiplying the coordinates of P by the scale factor $P(x\ \ y) \xrightarrow{\text{Dilation } (n) \text{ factor}} (nx, ny)$

15. $A'B'$ is dilation image of AB.
What is the scale factor of the dilation n,
if $AB = 6$, and $A'B' = 4$?

A $\frac{2}{3}$ **B** $\frac{3}{2}$

C 4 **D** 6

$$n = \frac{A'B'}{AB}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$ **A**

7.16 $A'B'$ is dilation image of AB. Find AB if the dilation scale factor is $n = \frac{1}{4}$, and $A'B' = 12$

A 3 **B** 48

C 24 **D** 12

$$n = \frac{A'B'}{AB}$$

$$\frac{1}{4} = \frac{12}{AB}$$

$$AB = 4 \times 12$$

$$= 48$$ **B**

17. Find the image of the point $(6,-8)$ for a dilation with scale factor $n=\frac{1}{2}$

A $(12,-16)$ **B** $(-3,4)$

C $(3,-4)$ **D** $(6,-8)$

$$P(x,y)\xrightarrow{\text{Dilation }(n)\text{ factor}}(nx,ny)$$

$$P\ (6,-8)\xrightarrow{n=\frac{1}{2}}\left(\frac{1}{2}\cdot 6,\ \frac{1}{2}(-8)\ \right)$$

$(3,\ -4)$ ⇒ **C**

18. Which transformation is not isometry.

A Translation **B** Reflection

C Rotation **D** Dilation

In **Dilation** the image and the preimage are not congruent. ⇒ **D**