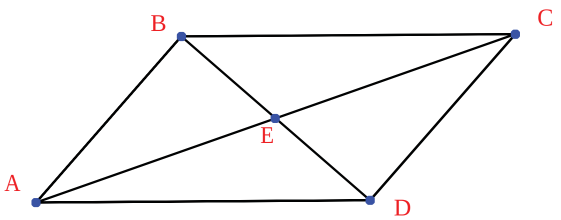
CHAPTER (6)

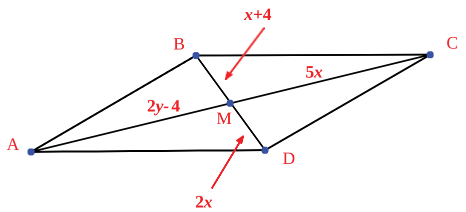
QUADRILATERALS

Properties of Parallelograms

- The opposite sides are congruent
  $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
- The consecutive angles are supplementary
  $m\angle A + m\angle B = 180$
- The opposite angles are congruent
  $\angle A \cong \angle C$ and $\angle B \cong \angle D$
- The diagonals bisect each other
  $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$



1. Find the length of $\overline{AC}$ in the adjacent parallelogram



A 20 **B** 40
C 30 **D** 50

The diagonals bisect each other

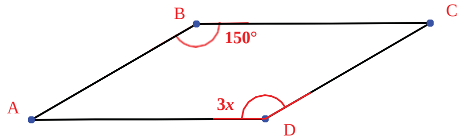
$BM = MD$	$MC = 5x$
$x + 4 = 2x$	$= 5 \times 4 = 20$
$4 = x$	

We can solve the equation $2y - 4 = 20$ and find AM then add it to AC, but this is a long method

$$AC = AM + MC$$
$$= 2AM \qquad AM = MC$$
$$= 2 \times 20 = 40$$

➤ **B**

2. Find x in the parallelogram



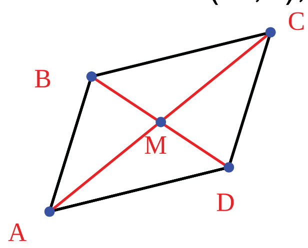
A 50 **B** 100
C 25 **D** 125

Opposite angles are congruent

$$3x = 150$$
$$x = 50$$

➤ **A**

3. Find the coordinates of the point M in the parallelogram $ABCD$ if $A(-6,-4)$, B(-4,2),C(4,4),D(2,-2)



A $(-1,0)$ **B** $(1,0)$
C $(0,-1)$ **D** $(0,1)$

The diagonals bisect each other; therefore, M is a midpoint for both diagonals

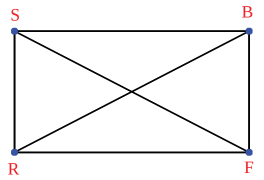
$$M_{\overline{AC}} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-6+4}{2}, \frac{-4+4}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{0}{2}\right) = (-1, 0)$$

➤ **A**

Special Parallelograms

- A **rhombus** is a parallelogram with four congruent side and perpendicular diagonals
- A **rectangle** is parallelogram with four right angles and congruent diagonals
- A **square** is a parallelogram with four congruent sides and four right angles and congruent, perpendicular diagonals

4. In the rectangle $RSBF$, $SF = 2x + 15$ and $RB = 5x - 12$, What is the length of the diagonal?



A 1 **B** 9

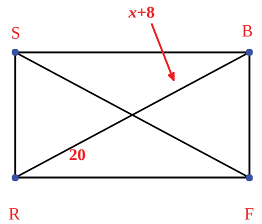
C 18 **D** 33

The diagonals of a rectangle are congruent

$SF = RB$
$2x + 15 = 5x - 12$
$27 = 3x$
$x = 9$

$RB = 5x - 12$
$= 5 \times 9 - 12$
$= 45 - 12$
$= 33$ ➤ **D**

5. In the rectangle, find x



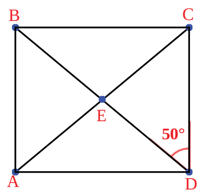
A 24 **B** 28

C 12 **D** 14

The diagonals of a rectangle are congruent and bisect each other

$x + 8 = 20$
$x = 12$ ➤ **C**

6. In the rectangle $ABCD$, find the $m\angle AED$



A 110 **B** 100

C 50 **D** 80

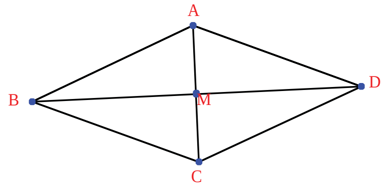
The diagonals of a rectangles are congruent and bisect each other

$EC = ED \rightarrow \Delta ECD$ $\Delta$is an isosceles triangle,
$\rightarrow m\angle ECD = 50$

$\angle AED$ is a remote angle

$m\angle AED = m\angle EDC + m\angle ECD$
$= 50 + 50 = 100$ ➤ **B**

7. In the rhombus $ABCD$ if $AC = 10$ and $BD = 24$, then find the side length of the rhombus



A 5 **B** 13

C 12 **D** 8

The diagonals are perpendicular and bisect each other

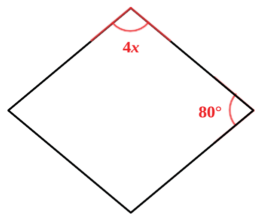
$AM = \frac{AC}{2} = \frac{10}{2} = 5$

$DM = \frac{BD}{2} = \frac{24}{2} = 12$

Using Pythagorean triple $(5, 12, 13)$ in any triangle that contains two halves of the diagonals, let's consider the triangle AMD then the hypotenuse is the side of the rhombus

➤ **B**

8. Find x in the rhombus at the right



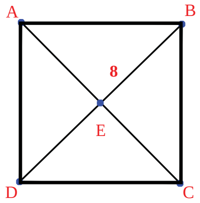
A 80 **B** 100

C 20 **D** 25

The rhombus is a parallelogram
$\rightarrow$ Consecutive angles are supplementary

$4x + 80 = 180$
$4x = 100$
$x = 25$ ➤ **D**

9. In the square $BE = 8$, find AC



A 16 **B** 8
C 12 **D** 24

The diagonals of the square are congruent and bisect each other $BD = 2BE$

$$= 2 \times 8$$

$$= 16$$

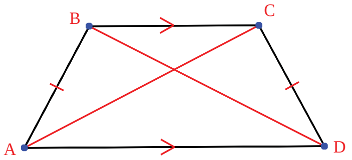
$$AC = BD = 16$$ ➤ **A**

10. If the diagonals of a quadrilateral are perpendicular then the shape is square or

A Rectangle **B** Rhombus
C Parallelogram **D** Trapezoid

By the properties of rhombus ➤ **B**

Trapezoid



Trapezoid is a quadrilateral with exactly one pair of parallel sides

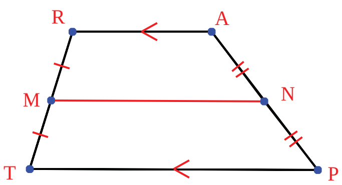
- An isosceles trapezoid is a trapezoid with legs that are congruent $\overline{AB} \approx \overline{CB}$
- Each pair of base angles is congruent
  $\angle BAD \approx \angle CDA, \ \angle ABC \approx \angle DCB$
- The diagonals are congruent $\overline{AC} \approx \overline{DB}$

Trapezoid Mid-segment

- The mid-segment is parallel to the bases

$\overline{MN} \parallel \overline{TP}$

$\overline{MN} \parallel \overline{RA}$

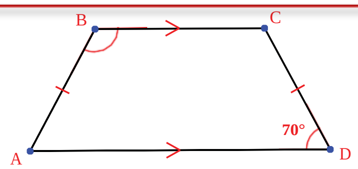


- The length of the mid-segment is half the sum of the lengths of the bases

$$M_{segment} = \frac{b_1 + b_2}{2}$$

$$MN = \frac{TP + RA}{2}$$

11. Find $m\angle B$



A 100 **B** 110
C 70 **D** 120

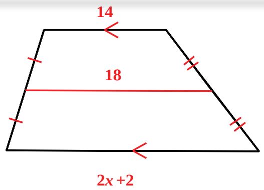
Base angles are congruent in an isosceles trapezoid
$\to m\angle A = 70$

$m\angle B + m\angle A = 180$

$m\angle B + 70 = 180$

$m\angle B = 110$ ➤ **B**

12. Find x



A 10 **B** 18
C 20 **D** 22

Trapezoid mid-segment

$$M_{segment} = \frac{b_1 + b_2}{2}$$

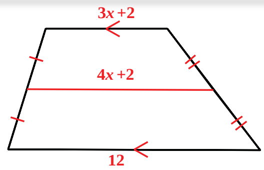
$$18 = \frac{14 + (2x + 2)}{2}$$

$$18 = \frac{2x + 16}{2}$$

$$18 = x + 8$$

$$10 = x$$ ➤ **A**

13. Find x



A 10 **B** 5
C 2 **D** 20

Trapezoid mid-segment

$$M_{segment} = \frac{b_1 + b_2}{2}$$

$$4x + 2 = \frac{3x + 2 + 12}{2}$$

$$2(4x + 2) = 3x + 14$$

$$8x + 4 = 3x + 14$$

$$5x = 10$$

$$x = 2$$ ➤ **C**

Similar Polygons

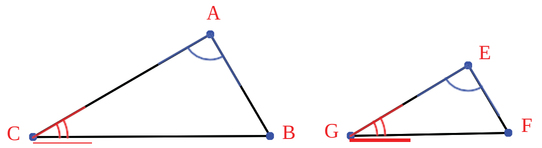
Two polygons are similar polygons if the corresponding angles are congruent and if the lengths of the corresponding sides are proportional

Similar Triangles

Angle-Angle similarity $AA\sim$

Two angles are congruent to two angles of another triangle

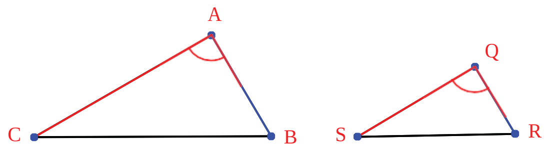
$$\angle A \cong \angle E, \text{ and } \angle C \cong \angle G$$
$$\rightarrow \Delta ABC \sim \Delta EFG$$



Side-Angle-Side $SAS\sim$

One angle of one triangle is congruent to an angle of the second triangle and sides that include the two angles are proportional.

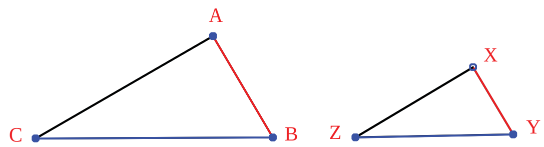
$$\angle A \cong \angle Q, \text{ and } \frac{AB}{QR} = \frac{AC}{QS}$$
$$\rightarrow \Delta ABC \sim \Delta QRS$$



Side –Side-Side $SSS\sim$

The corresponding sides of two triangles are proportional.

$$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{YZ}{BC}$$
$$\rightarrow \Delta ABC \sim \Delta XYZ$$



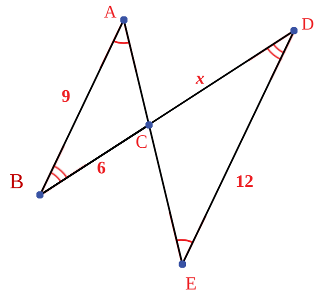
14. If $\Delta ABC \sim \Delta WXY$, then...

A $\angle A \cong \angle W$ **B** $\overline{BC} \cong \overline{WY}$

C $\angle B \cong \angle Y$ **D** $\overline{AB} \cong \overline{YX}$

If two triangles are similar, then corresponding angles are congruent $\angle A \cong \angle W$ ➤ **A**

15. The polygons are similar. Find the value of x



A 6 **B** 8

C 12 **D** 24

Scale factor $= \frac{AB}{DE} = \frac{9}{12} = \frac{3}{4}$

$$\frac{AB}{DE} = \frac{BC}{DC}$$
$$\frac{3}{4} = \frac{6}{x}$$
$$3x = 4 \times 6$$
$$x = \frac{4 \times 6}{3}$$
$$= 8$$

➤ **B**

Area and perimeter of similar Triangles

If the similarity ratio of two similar figures is $\frac{a}{b}$

then the ratio of their perimeters is $\frac{a}{b}$

and the ratio of their areas is $\left(\frac{a}{b}\right)^2$

16. If two triangles are similar, their perimeters are: 32 and 24, and the length of the side of the bigger triangle is **8**, find the length of the corresponding side in the other triangle

A 6 **B** 4

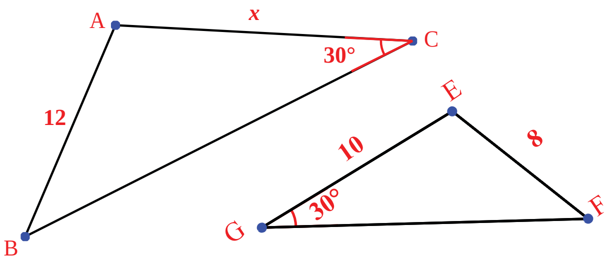
C 10 **D** 8

The scale factor of similar triangles is the same as the ratio of their perimeters.

$$\frac{P\Delta_1}{P\Delta_2} = \frac{S_1}{S_2}$$
$$\frac{32}{24} = \frac{8}{x}$$
$$x = \frac{8 \times 24}{32}$$
$$= 6$$

➤ **A**

17. If the two triangles are similar then find x



A 16 **B** 8

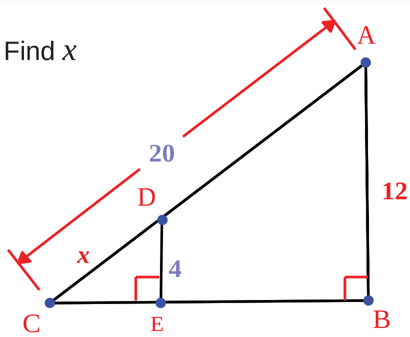
C 15 **D** 50

Since the two triangles are similar then their corresponding sides are proportional

$$\frac{8}{12}=\frac{10}{x}$$
$$x=\frac{10\times12}{8}$$
$$=15$$

➤ **C**

18. Find x



A 4 **B** 21

C 14 **D** 7

$\angle C \cong \angle C$ **Reflexive property**

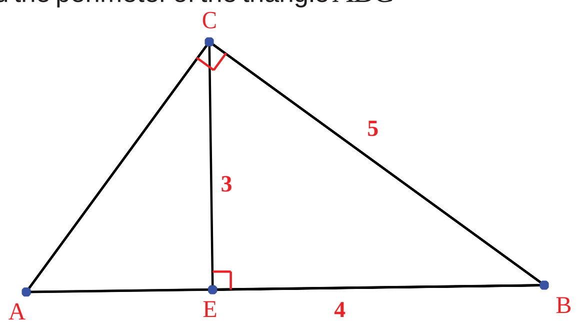
$\angle CED \cong \angle CBA = 90°$

$\Delta CBD \cong \Delta CED$ by $AA\sim$

$$\frac{4}{12}=\frac{x}{20}$$
$$x=\frac{4\cdot20}{12}$$
$$x\approx7$$

➤ **D**

19. Find the perimeter of the triangle ABC



A 30 **B** 15

C 24 **D** 36

$m\angle C = m\angle BEC = 90$

$\overline{CB}\cong\overline{CB}$ Reflexive Propor

$\angle B\cong\angle B$ Reflexive Propor

$\rightarrow \Delta ABC \sim \Delta CEB$

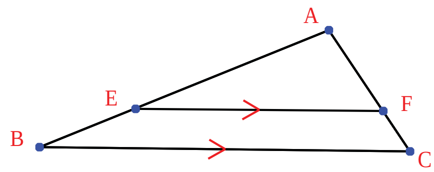
$$\frac{CB}{EB}=\frac{P\Delta ABC}{P\Delta CEB}$$
$$\frac{5}{4}=\frac{x}{3+4+5}$$
$$\frac{5}{4}=\frac{x}{12}$$
$$4x=5\times12$$
$$x=15$$

➤ **B**

Side Splitter Theorem

If $\overline{EF}\parallel\overline{BC}$,

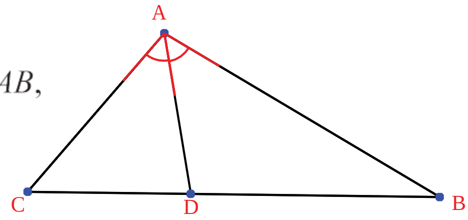
then $\dfrac{BE}{EA}=\dfrac{CF}{FA}$



Triangle-Angle-Bisector-Theorem

If $\overline{AD}$ bisects $\angle CAB$,

then $\dfrac{CD}{DB}=\dfrac{CA}{BA}$

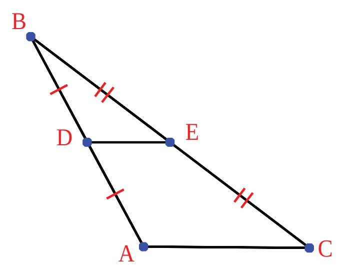


Tringle Mid-Segment-Theorem

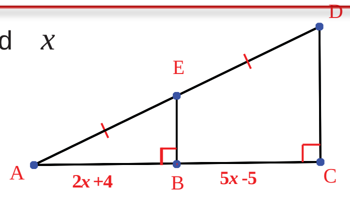
If $\overline{DE}$ Bisects $\overline{AB}$ and $\overline{CB}$,

then $\rightarrow \overline{DE}\parallel\overline{AC}$

$\rightarrow DE=\dfrac{AC}{2}$



20. Find x



A 3 **B** 4

C 9 **D** 12

$\overline{EB} \parallel \overline{DC}$

$\overline{EB}$ bisects $\overline{AD}$

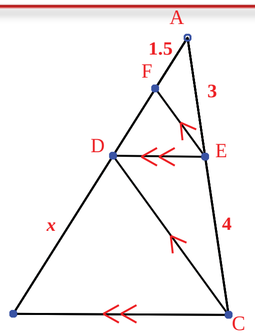
Therfore by the Tringle Mid-Segment-Theorem

$\overline{EB}$ bisects $\overline{AC}$

$$AB = BC$$
$$2x + 4 = 5x - 5$$
$$9 = 3x$$
$$3 = x$$

➤ **A**

21. Find x



A $\frac{3}{14}$ **B** $\frac{17}{3}$

C $\frac{14}{3}$ **D** $\frac{3}{17}$

In the ΔACD

$\overline{FE} \parallel \overline{DC}$

$$\frac{AE}{EC} = \frac{AF}{FD}$$
$$\frac{3}{4} = \frac{1.5}{FD}$$
$$FD = 2$$

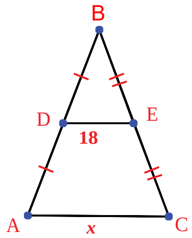
In the ΔACB

$\overline{DE} \parallel \overline{BC}$

$$\frac{AE}{EC} = \frac{AD}{DB}$$
$$\frac{3}{4} = \frac{1.5 + 2}{x}$$
$$x = \frac{14}{3}$$

➤ **C**

22. Find x



B

A 9 **B** 16

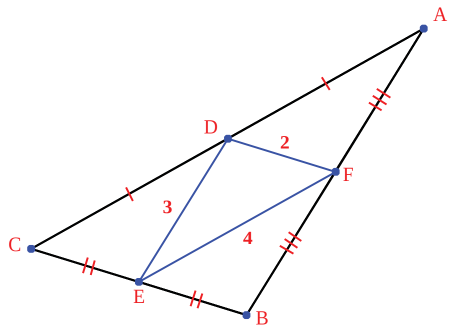
C 24 **D** 36

Using Tringle Mid-Segment-Theorem

$$18 = \frac{x}{2}$$
$$x = 36$$

➤ **D**

23. Find the perimeter of the triangle ABC



A 16 **B** 18

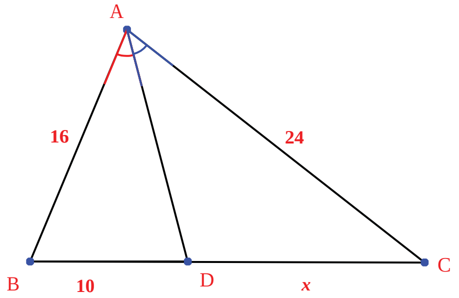
C 14 **D** 15

$\overline{DE}$, $\overline{FD}$, $\overline{FE}$ are sides bisectors in ΔABC

$$AB = 2DE = 2 \times 3 = 6$$
$$BC = 2FD = 2 \times 2 = 4$$
$$AC = 2FE = 2 \times 4 = 8$$
$$P\Delta ABC = AB + BC + AC$$
$$= 6 + 4 + 8$$
$$= 18$$

➤ **B**

24. Find x



A 30 **B** 15

C 24 **D** 36

Use angle bisector theorem

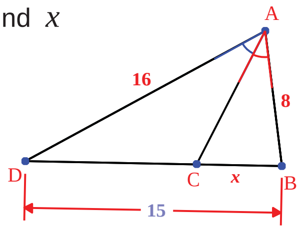
$$\frac{16}{10}=\frac{24}{x}$$

$$x=\frac{24\times 10}{16}$$

$$=15$$

➤ **B**

25. Find x



A 5 **B** 15

C 8 **D** 24

$$DC=DB-CB$$

$$=15-x$$

Using angle bisector theorem

$$\frac{AD}{CD}=\frac{AB}{CB}$$

$$\frac{16}{15-x}=\frac{8}{x}$$

$$16x=8(15-x)$$

$$2x=15-x$$

$$3x=15$$

$$x=5$$

➤ **A**