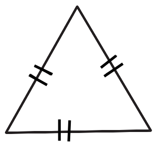
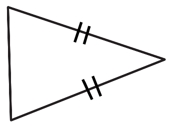
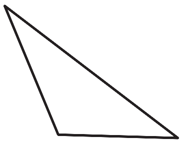
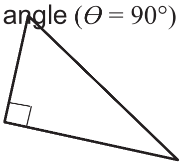
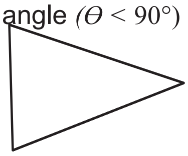
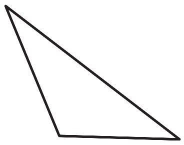
CHAPTER (5)

# TRIANGLES AND POLYGONS

## Triangle types

| Classifying triangles by their sides | | |
|---|---|---|
| **Equilateral**<br>Has three sides that have the same length<br> | **Isosceles**<br>Has at least two sides that have the same length<br> | **Scalene**<br>Has no sides that have the same length<br> |
| **Classifying triangles by their angles** | | |
| **Right**<br>Has one angle that is a right angle ($\theta = 90°$)<br> | **Acute**<br>Has all angles that are a acute angle ($\theta < 90°$)<br> | **Obtuse**<br>Has one angle that is an obtuse angle ($\theta > 90°$)<br> |

## Conclusion

- The base angles of an isosceles triangle are congruent
- The equilateral triangle is also equiangular and each angle measure's $60^\circ$
- The sum of the angles of any triangle is $180^\circ$
- If an isosceles triangle has a $60^\circ$ angle then it is an equilateral triangle

**1. Classify the triangle if it has $64^\circ$ $64^\circ$ $52^\circ$ angles**

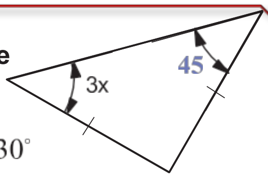
A Right triangle B Obtuse triangle

C Equilateral triangle D Isosceles triangle

Since the base angle are congruent then it is an isosceles triangle

D

**2. Find $x$ in the adjacent triangle**




A $45^\circ$ B $30^\circ$

C $15^\circ$ D $90^\circ$

Given that two sides are congruent

→ the base angles are congruent

$3x = 45$

$x = 15$

C

**3. If the measure of the obtuse angle '$A$' in the isosceles triangle $ABC$ is $100^\circ$ then the measure of one of the other two angles is**

A $80^\circ$ B $50^\circ$

C $40^\circ$ D $100^\circ$

In the isosceles triangle $ABC$

Since the obtuse angle is $A$ the base angles are $\angle B$ and $\angle C$, and they are congruent

$\angle A + \angle B + \angle C = 180$

$\angle A + 2\angle B = 180$

$100 + 2\angle B = 180$

$2\angle B = 80$

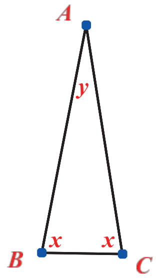
$\angle B = 40$

C

**4. In the triangle, if $x = 4y$, then find $x$**

A 80 B 20

C 40 D 160




$x + x + y = 180$

$4y + 4y + y = 180$

$9y = 180$

$y = 20$

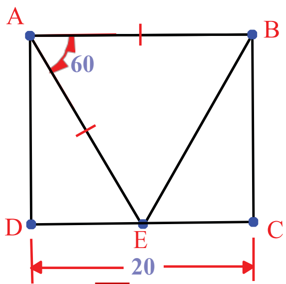
$\rightarrow x = 4y$

$= 4 \times 20$

$\rightarrow x = 80$

A

**5. $ABCD$ is a rectangle, find the perimeter of the triangle $ABE$**




A 40 B 20

C 60 D 80

In the rectangle $AB = CD = 20$

Since that the isosceles triangle has one angle that measures $60^\circ$ then it is an equilateral triangle

$20 + 20 + 20 = 60$

C

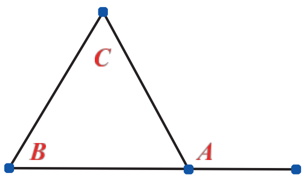
## Exterior Angle

An exterior angle of a triangle is formed by extending one of the sides. Each exterior angle has two remote interior angles.

$\angle A =$ Exterior angle

$\angle B$ and $\angle C$ are remote interior angles

Remote angles formula states that the measure of an exterior angle $\angle A$ equals the sum of the remote interior angles




$$\angle Exterior = \angle I_1 + \angle I_2$$
$$\angle A = \angle B + \angle C$$

**6. Find the exterior angle of an equilateral triangle**

A $100$ B $90$

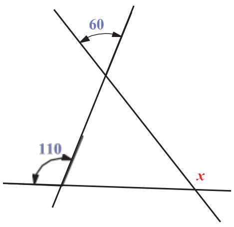
C $60$ D $120$

Hence the measure of all angles of the equilateral triangle are $60^o$ too.

$$\angle E = \angle Interior_1 + \angle Interior_2$$
$$= 60 + 60$$
$$= 120$$

**D**

**7. Find the measure of angle $x$**



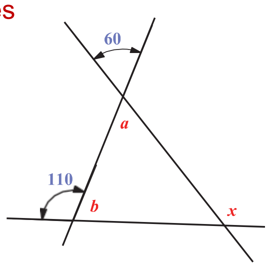

A $60^\circ$ B $130^\circ$

C $70^\circ$ D $120^\circ$

$\angle a = 60$ vertical angles

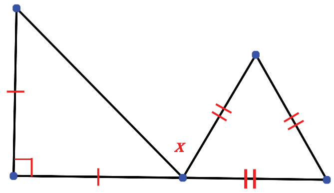
$\angle b = 70$ linear pair angles

$$\angle E = \angle I_1 + \angle I_2$$
$$= 60 + 70$$
$$= 130$$




**B**

**8. Find $x$ in the figure**



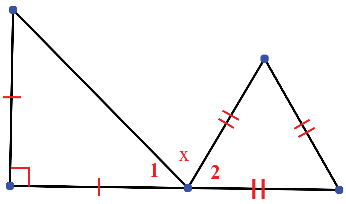

A $60^o$ B $45^o$

C $105^o$ D $75^o$

$\angle_1 = \frac{180 - 90}{2} = 45$ isosceles triangle

$\angle_2 = 60$ equilateral triangle

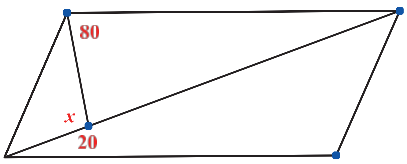
$\angle_1 + x + \angle_2 = 180$ adjacent angles on a line




$$45 + x + 60 = 180$$
$$105 + x = 180$$
$$x = 75$$

**D**

**9. Find $x$ in the parallelogram**

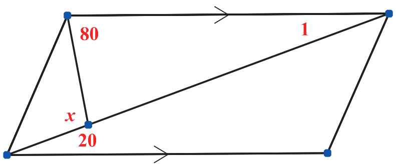



A $80$ B $100$

C $60$ D $120$

$\angle_1 = 20$ Alternate interior

$\angle x = 80 + 20 = 100$ Remote angle

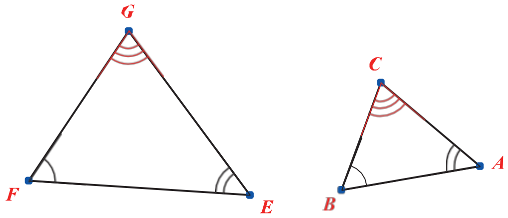



**B**

## Congruent Polygons

Congruent Polygons have congruent corresponding parts (their matching sides and angles)

## Third Angle Theorem



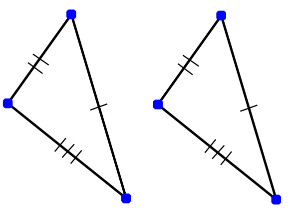
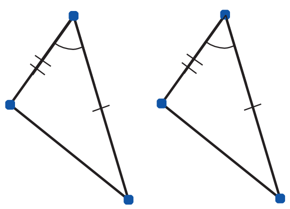
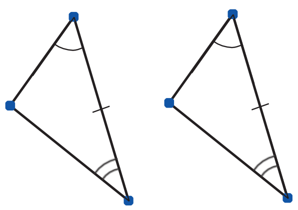
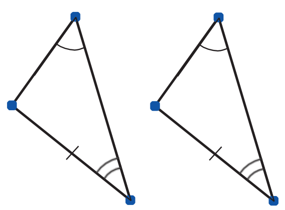

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

$\angle A \cong \angle E$

$\angle B \cong \angle F$

$\rightarrow \angle C \cong \angle G$

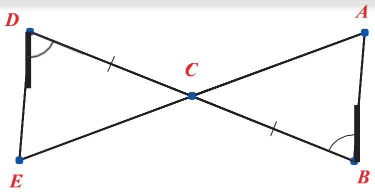
## Triangle Congruence Theorems

| | |
|---|---|
|  |  |
| **SSS (side-side-side)** All three corresponding sides are congruent | SAS (side-angle-side) Two sides and the angle between them are congruent |
|  |  |
| ASA (angle-side-angle) Two angles and the side included between them are congruent | AAS (angle-angle-side) Two angles and non-included side are congruent |

**10.** **State the theorem you can use to prove that the triangles are congruent**

**A** SSS **B** SAS

**C** ASA **D** AAS

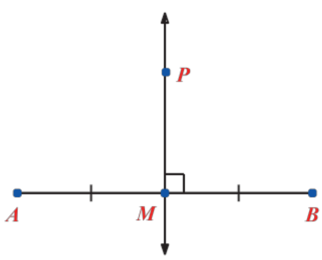
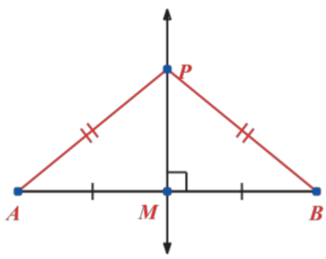



| | | |
|---|---|---|
| $\angle D \cong \angle B$ | given | → A |
| $\angle BCA \cong \angle DCE$ | vertical angles | → A |
| $BC = DC$ | given | → S |
| | | ASA |

→ C

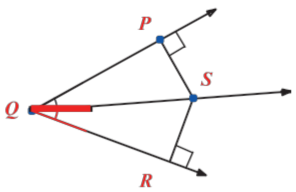
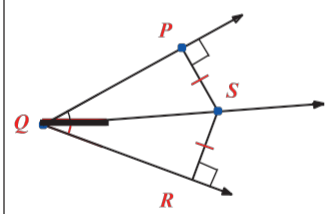
## Perpendicular Bisector

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment

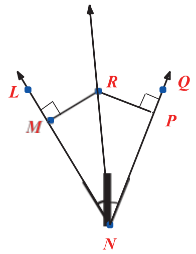
| If | Then |
|---|---|
| $PM \perp AB$<br>$MA = MB$ | $PA = PB$ |
|  |  |

## Angle Bisector

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle

| If | Then |
|---|---|
| $\overrightarrow{QS}$ bisects $\angle PQR$<br>$\overline{SP} \perp \overrightarrow{QP}$,<br>$\overline{SR} \perp \overrightarrow{QR}$ | $SP = SR$ |
|  |  |

**11. If length of $RM = 7x$ and $RP = 2x + 25$, what is $\overline{RM}$**




A 5 B 35

C 25 D 10

$RM = RP$

$7x = 2x + 25$

$5x = 25$

$x = 5$

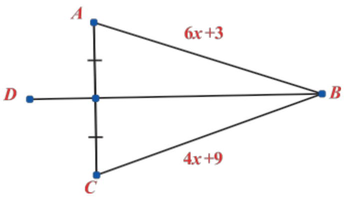
$RM = 7x$

$= 7(5)$

$= 35$

**B**

**12. If $\overline{BD}$ is the perpendicular bisector, find x**




A 3 B 6

C 9 D 12

$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$ so $B$ is equidistant from $A$ and $C$

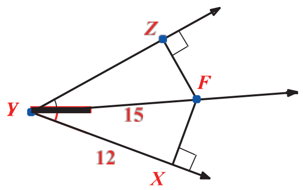
$6x + 3 = 4x + 9$

$2x = 6$

$x = 3$

**A**

**5.13 If $FY = 15$, $XY = 12$, then find the length of $\overline{FZ}$**




A 9 B 12

C 15 D 18

Since $\overrightarrow{YF}$ bisects $\angle XYZ$

$XF = FZ$

In $\Delta YXF$ use the Pythagorean triple:

$3(3,4,5) \rightarrow (9,12,15)$, therefore $XF = FZ = 9$

**A**

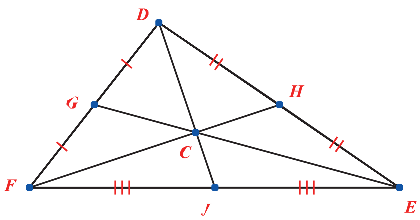
## Median Theorem

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side correct

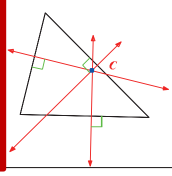
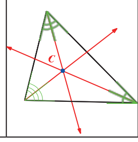
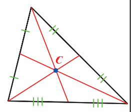
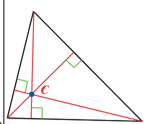
$DC = \frac{2}{3} DJ$

$EC = \frac{2}{3} EG$

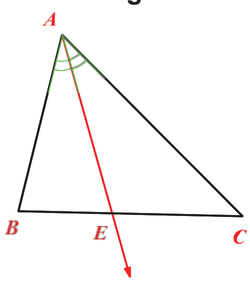
$FC = \frac{2}{3} FH$




## Special Segments and Lines in Triangles

| Special | Perpendicular Bisectors | Angle Bisectors | Medians | Altitudes |
|---|---|---|---|---|
| The point C | Circumcenter | Incenter | Centroid | Orthocenter |
| Triangle |  |  |  |  |

**5.14 What is $\overline{AE}$ in the triangle $ABC$**

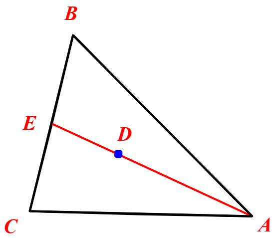



A Perpendicular Bisector B Angle bisector

C Median D Altitude

**B**

**5.15** **If $D$ is the centroid of the triangle $ABC$ and $AE=18$, then $DA = \dots$**




**A** 8 **B** 18

**C** 12 **D** 15

$$DA = \frac{2}{3}AE$$

$$= \frac{2}{3} \times 18$$

$$= 12$$

➢ **C**

**5.16** In the diagram, $XA = 8$, What is length of $\overline{XB}$?

**A** 15 **B** 18

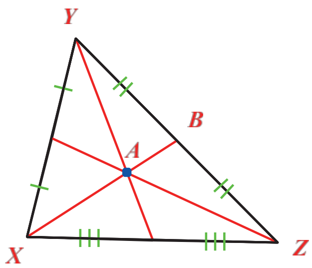
**C** 12 **D** 6

$$XA = \frac{2}{3}XB$$

$$8 = \frac{2}{3}XB$$

$$(\frac{3}{2})8 = (\frac{3}{2})\frac{2}{3}XB$$

$$12 = XB$$



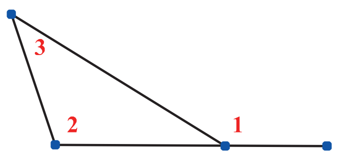

➢ **C**

## Triangle Inequality Theorems

Triangle Exterior Angle Theorem

$m\angle_1 > m\angle_2$

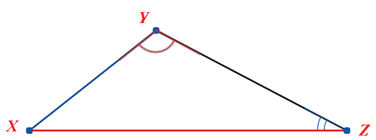
$m\angle_1 > m\angle_3$




- In a triangle, if two sides of a triangle are not congruent, then the larger angle lies opposite the longer side and vice versa

If $XZ > XY \leftrightarrow Then$

$m\angle Y > m\angle Z$

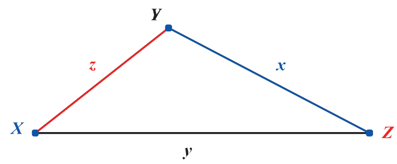



- The sum of the lengths of any two sides of a triangle is greater than the length of the third side

$x + z > y$

$x + y > z$

$y + z > x$




- The difference of the lengths of any two sides of a triangle is less than the length of the third side

$|x - z| < y$

$|x - y| < z$

$|y - z| < x$

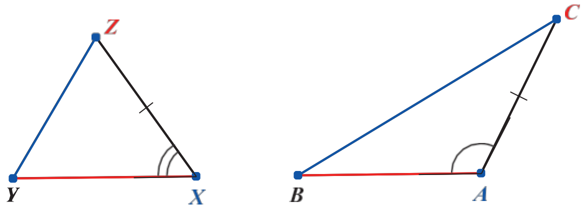
for any side x in a triangle $XYZ$

$|y - z| < x < y + z$

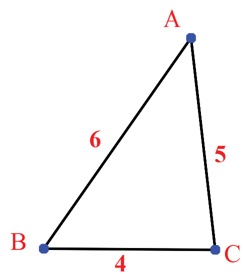
- SAS Inequality Theorem

if $\overline{AC} \cong \overline{XZ}$ , $\overline{AB} \cong \overline{XY}$ and $m\angle A > m\angle X$

Then $BC > YZ$ and vice versa




**5.17** **In the adjacent triangle which statement is true?**




**A** $\angle A = \angle C$ **B** $\angle A > \angle B$

**C** $\angle B < \angle C$ **D** $\angle B = \angle C$

Since $\angle B$ lies opposite a side 5 less than the $\angle C$ which lies opposite a side 6

$\angle B < \angle C$

➢ **C**

**5.18 Can a triangle have sides with the given lengths?**

A $3,7,8$ B $5,10,15$
C $2,5,8$ D $4,6,10$

Add the least two sides and compare them to the third side they should satisfy

$a+b>c$

$3+7>8$

$10>8$

A

**5.19 If two sides of a triangle are $5$ and $8$ then which is the least possible integer value of the length for the third side?**

A $3$ B $13$
C $4$ D $14$

For the side x

$|y-z|<x<y+x$

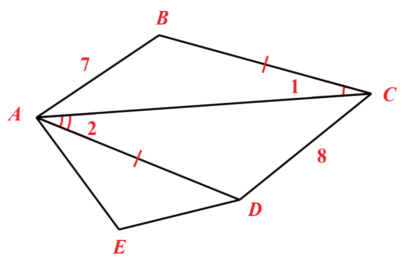
$|8-5|<x<8+5$

$3<x<13$

Only option $C$ satisfies this inequality

C

**5.20 Compare** $m\angle 1$ to $m\angle 2$




A < B >
C = D ≥

Since $AD=BC$ and $AC=AC$

$\angle 1$ lies opposite side $7$

$\angle 2$ lies opposite side $8$

$\angle 1<\angle 2$

A

## Polygons

- The sum of the measures of the interior angles of an n-gon is $S=(n-2)180$
- The measure of each interior angle of a regular n-polygon is $m=\dfrac{(n-2)180}{n}$
- A regular polygon is a polygon with all sides and all angles congruent
- The number of sides of a regular polygon that has interior angle $m$ is $n=\dfrac{360}{180-m}$
- The sum of the measures of exterior angles of a polygon one at each vertex, is $360$
- The measure of an exterior angle of a regular polygon is $\dfrac{360}{n}$

**5.21 Find the some of the interior angle measures of a heptagon**

A $720$ B $540$
C $900$ D $1080$

$S=(n-2)180$

$=(7-2)180$

$=(5)180$

$=900$

C

**5.22 What is the measure of each interior angle in a hexagon**

A $120$ B $72$
C $108$ D $90$

$m=\dfrac{(n-2)180}{n}$

$=\dfrac{(6-2)180}{6}$

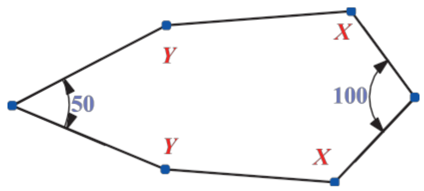
$=\dfrac{(4)180}{6}$

$=4\times 30$

$=120$

A

**5.23** **Find** $x+y$




A 108
B 195
C 720
D 285

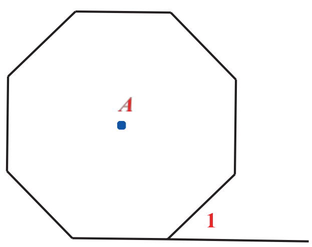
The polygon is hexagon

$S=(n-2)180$
$=(6-2)180$
$=720$

$x+x+y+y+50+100=720$
$2x+2y+150=720$
$2x+2y=570$
$2(x+y)=570$
$x+y=285$

D

**5.24** **What is** $m\angle 1$ **in the regular octagon**




A 72
B 45
C 60
D 135

Exterior angle $=\dfrac{360}{n}$
$=\dfrac{360}{8}$
$=45$

B

**5.25** **Find the number of sides of a regular polygon that has an interior angle measures** $140^{\circ}$

A 9
B 8
C 7
D 10

$n=\dfrac{360}{180-m}$
$=\dfrac{360}{180-140}$
$=\dfrac{360}{40}$
$=9$

A