CHAPTER (3)

SEQUENCE AND SERIES

1- Sequences

A sequence is a set of number that are in order. Each number in the sequence is called a term.

A- Arithmetic sequence (ASq)

In an arithmetic sequence the difference between one term and the next term is constant and denoted by 'd'. In other words, we just add the same value each time.

Example

$1, 4, 7, 10, 13\ldots$

This sequence has a difference $d = 3$ between each two terms. The pattern is continued by adding 3 to the last number each time

$1 \quad 4 \quad 7 \quad 10 \quad 13 \quad \ldots n^{th}$ term

$+3 \quad +3 \quad +3 \quad +3 \quad +3$

To find the n^{th} term in the ASq we can use the rule $a_n = a_1 + d(n-1)$ where 'a_n' is 'n^{th}' term, 'a_1' is the first term 'n' is the number of term and 'd' is the difference

Arithmetic Mean

The arithmetic means are the terms between two non-consecutive terms for example in the ASq $4, 9, 14, 19, 24$, the terms $9, 14, 19$ are three arithmetic means between 4 and 24.

On the other hand if (a, b, c) are three consecutive terms in a ASq, then the arithmetic mean: $b = \frac{a+c}{2}$

1. Find the common difference of the ASq $10, 7, 4, 1, -2, \ldots$

A 3 B -3

C -12 D 12

$d = a_2 - a_1$
$= 7 - 10$
$= -3$ ⇒ **B**

2. Find the value of x in the ASq $8, 2, x, -10\ldots$

A 4 B 6

C -4 D -6

$d = a_2 - a_1$
$= 2 - 8$
$= -6$

$x - 2 = d$

$x - 2 = -6$

$x = -4$ ⇒ **C**

3. Find the next term in the ASq $\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}$

A $\frac{1}{4}$ B $\frac{6}{4}$

C $\frac{7}{4}$ D $\frac{-7}{4}$

$d = 1 - \frac{3}{4}$
$= \frac{4}{4} - \frac{3}{4}$
$= \frac{1}{4}$

Next term

$\frac{3}{2} + \frac{1}{4}$

$\frac{6}{4} + \frac{1}{4} = \frac{7}{4}$

⇒ **C**

4. The value of the company's share is 80 riyals, and after 3 months its value becomes 92 riyals. If the value of the share is in form of arithmetic sequence , then find its value after 7 months.

A 104 **B** 108

C 92 **D** 90

The increment of share through the 3 months is $92-80=12$

To find the incremen in each month we divide

$\frac{12}{3}=4$

$n=7 \quad d=4 \quad a_1=80$

$a_7=a_1+(n-1)d$

$=80+(7-1)\cdot 4$

$=80+24$

$=104$ ⋙ **A**

5. Find the common difference in the ASq if

$a_9=-12,\ a_1=20$

A -6 **B** 6

C 4 **D** -4

$a_9=-12 \quad a_1=20 \quad n=9 \quad d=?$

$a_n=a_1+(n-1)d$

$-12=20+(9-1)d$

$-12=20+8d$

$-32=8d$

$d=-4$

⋙ **D**

B- Geometric Sequence $(\ GSq\ )$

In a geometric sequence each term is found by multiplying the previous term by a constant called common ratio and denoted by r.

Example

$1, 2, 4, 8, 16, \ldots$

This sequence has a common ratio $r=2$ between each two terms. In other words, each term is found by multiplying the previous term by 2.

$1 \quad 2 \quad 4 \quad 8 \quad 16 \quad \ldots$ n^{th} term

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2 \quad \times 2$

To find the n^{th} term in the GSq we use the rule $a_n=ar^{n-1}$ where a_n is the n^{th} term 'a_1 ' is the first term.' n is the number of terms and r is the common ratio.

Geometric Mean

The geometric mean are terms between two non-consecutive terms for example in the GSq $-2,\ 4,\ -8,\ 16, -32$ the terms $4,\ -8,\ 16$ are three geometric means between -2, and -32. On the other hand if $\{a,b,c\}$ are three consecutive terms in a GSq then the geometric mean is $\sqrt{a\,c}$

6. Find the common ratio of the GSq

$2,\ -6,\ 18,\ -54$

A 3 **B** -3

C 2 **D** -2

$r=\frac{a_2}{a_1}$

$=\frac{-6}{2}=-3$

⋙ **B**

7. If $a>1$ then which of the following is a GSq

A $a^2,\ a^4,\ a^6,\ a^8,\ldots$ **B** $a,\ 2a,\ 3a,\ 4a,\ldots$

C $a^2+1,\ a^2+2,\ a^2+3,\ldots$ **D** $a+1,\ a-2,\ a+3,\ldots$

he geometric sequence has a common ratio by dividing

$\frac{a_2}{a_1}=r \quad \frac{a^4}{a^2}=a^2,\ \frac{a^6}{a^4}=a^2,\ \frac{a^8}{a^6}=a^2$

⋙ **A**

8. Find the 4^{th} term in the GSq where

A 27 B 9 C 3 D 81

$a_1 = 1 \quad r = 3$

$n = 4 \quad a_1 = 1 \quad r = 3$

$a_n = a_1 r^{n-1}$

$a_4 = 1 \times 3^{4-1}$

$= 3^3$

$= 27$ ➤ A

9. The sum of the first 3 GSq terms is 28 and the sum of the next 3 terms is 224 find its ratio.

A 8 B 4 C 2 D -4

$a_2 = ra_1$

$a_3 = ra_2 = r \times ra_1 = r^2 a_1$

$a_1 + a_2 + a_3 = 28$

$a_1 + ra_1 + r^2 a_1 = 28$

$a_1 \left(1 + r + r^2\right) = 28$

$a_4 = ra_3 = r^2 a_1 \times r$

$= r^3 a_1$

$a_5 = a_4 r$

$= r^3 a_1 r$

$= r^4 a_1$

$a_6 = a_5 r$

$= r^4 a_1 r$

$= r^5 a_1$

$a_4 + a_5 + a_6 = 224$

$r^3 a_1 + r^4 a_1 + r^5 a_1 = 224$

$a_1 r^3 \left(1 + r + r^2\right) = 224$

$$\frac{a_1 r^3 \left(1 + r + r^2\right)}{a_1 \left(1 + r + r^2\right)} = \frac{224}{28}$$

$$\frac{r^3}{1} = 8$$

$\sqrt[3]{r^3} = \sqrt[3]{8}$

$= 2$ ➤ C

10. Find two geometric mean of the GSq $8, \ldots, \ldots, -64$

A $16, -32$ B $-16, 32$ C $16, 32$ D $-16, -32$

$a_n = a_1 r^{n-1}$

$a_4 = 8 \times r^3$

$-64 = 8 \times r^3$

$r^3 = -8 \longrightarrow r = -2$

$a_2 = a_1 r$

$= 8 \cdot -2$

$= -16$

$a_3 = a_2 \cdot r$

$= -16 \cdot -2$

$= 32$ ➤ B

11. Find the 5^{th} term in the GSq where $18, 12, 8, \ldots$

A $\frac{25}{12}$ B $\frac{25}{6}$ C $\frac{32}{9}$ D $\frac{23}{6}$

$$r = \frac{a_2}{a_1} = \frac{12}{18} = \frac{2}{3}$$

Method 1

$a_4 = 8 \cdot \frac{2}{3}$

$= \frac{16}{3}$

$a_5 = \frac{16}{3} \times \frac{2}{3}$

$= \frac{32}{9}$

Method 2

$a_n = a_1 r^{n-1}$

$a_5 = 18 \cdot \left(\frac{2}{3}\right)^4$

$= 18 \cdot \frac{16}{81}$

$= \frac{32}{9}$

➤ C

2- Series

The sum of sequence term is called series.

A- Arithmetic Series (ASr)

For a finite *ASr* that has ›*n*› terms, the first term is a_1 and last term is a_n, we can find the sum of series *Sn* by the rule:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Example

Find the sum of the first 10 terms of the *ASr* $7+9+11+\dots$

This sequence has a difference *d = 3* between each two terms. The pattern is continued by adding 3 to the last number each time

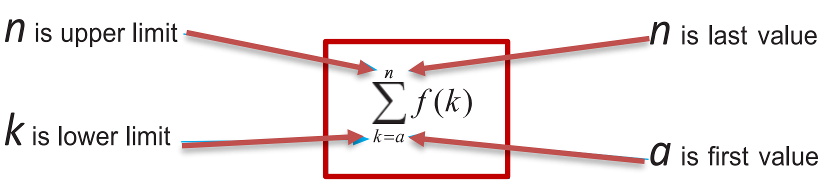
$$a_1 = 7 \qquad n = 10 \qquad d = 9-7 = 2$$

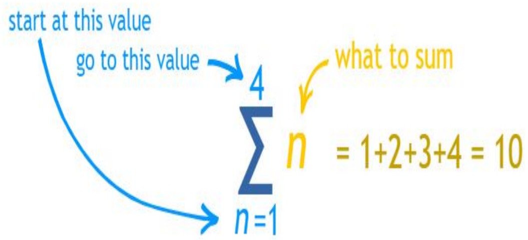
$$a_{10} = a_1 + (n-1)d = 7 + (10-1)2 = 25$$

$$S_{10} = \frac{n}{2}(a_1 + a_{10}) = \frac{10}{2}(7+25) = 5(32) = 160$$

Note that we can also use this rule: $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Sigma Notation $\sum$





- The number of terms in the series is found by: $n-a+1$
- $a_1 = f(firstTerm) = f(a)$
- $a_n = f(lastTerm) = f(n)$
- If $f(k)$ is a linear equation (first degree) then the $\sum_{k=a}^{n} f(k)$ is an AS_r and its common difference d is the coefficient of *k*

Example

Example Find the common difference and the number of terms of the $AS_r \sum_{k=5}^{20}(7k+3)$

The number of terms:

$n-a+1$

$20-5+1=16$

Common difference d is the coefficient of *k* $\to 7$

14. What is the number of terms of the $AS_r \sum_{k=6}^{17}(4k+9)$

A 11 B 10

C 12 D 9

Number of terms $= n-a+1 = 17-6+1 = 12$

➢ C

15. Find the first term of the $AS_r \sum_{k=3}^{20}(5k-1)$

A 14 B 15

C 19 D 13

$a_1 : k = 3 \to 5k-1 = 5(3)-1 = 14$

➢ A

16. Find the sum of the AS_r $\sum_{k=1}^{10}(4k-2)$

A 220 **B** 180

C 400 **D** 200

$a_1 \to k=1$

$a_1 = 4(1)-2$

$=2$

$a_n \to k=10$

$a_{10} = 4(10)-2$

$=38$

Number of terms

$=n-a+1$

$=10-1+1$

$=10$

$S_n = \frac{n}{2}(a_1+a_n)$

$= \frac{10}{2}(2+38)$

$=5(40)$

$=200$

D

17. Find the sum of the AS_r $\sum_{k=5}^{20}(2k-1)$

A 768 **B** 384

C 720 **D** 360

$a_1 \to k=5$

$a_1 = 2(5)-1$

$=9$

$a_n \to k=20$

$a_{20} = 2(20)-1$

$=39$

Number of terms

$=n-a+1$

$=20-5+1$

$=16$

$S_n = \frac{n}{2}(a_1+a_n)$

$= \frac{16}{2}(9+39)$

$=8(48)=384$

B

B- Geometric Series GS_r

For a finite GS_r that has ›n› terms, the first term is 'a_1' and the last term is 'a_n', we can find the sum of the series 'S_n' by the rule $S_n = \frac{a_1 - a_1 r^n}{1-r}$

Example

Find the sum of GS_r $\sum_{k=1}^{20} 3(2)^{k-1}$

$r=2$

$a_1 = 3(2)^{1-1}$

$=3(2)^0$

$=3$

$S_{20} = \frac{3-3\times 2^{20}}{1-2}$

$= \frac{3-3\times 2^{20}}{1-2}$

$= \frac{3(1-2^{20})}{-1}$

$= -3(1-2^{20})$

For an infinite GS_r / r / could be

- $|r| \geq 1$, the series is **diverge** and is the sum is infinity ∞
- $|r| < 1$, the series is **converge** and is the sum could be found by the rule $S = \frac{a_1}{1-r_1}$

Example

Find the sum $\sum_{k=1}^{\infty}(2)^{-k}$

Solution:

$\sum_{k=1}^{\infty}(2)^{-k} = \sum_{k=1}^{\infty}(\frac{1}{2})^k$ $r=\frac{1}{2}$ $a_1 = (\frac{1}{2})^1 = \frac{1}{2}$

The series is converging

$S = \frac{a_1}{1-r_1}$

$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

18. Find the sum $\sum_{n=1}^{11} 4(5)^{k-1}$

A $5^{10}-1$ **B** $5^{11}-1$

C $4^{10}-1$ **D** $4^{11}-1$

$a_1 = 4(5)^{1-1}$

$=4$

$r=5$

$S_n = \frac{a_1 - a_1 r^n}{1-r}$

$S_{11} = \frac{4-4\times 5^{11}}{1-5}$

$= \frac{4(1-5^{11})}{-4}$

$= -(1-5^{11})$

$=5^{11}-1$ B

19. If the *GSr* is a converge series then $|r|$ is

A $|r|>1$ **B** $|r|<1$

C $|r|\geq 1$ **D** $|r|\leq 1$

Since it is converge then $|r|<1$ ➤ **C**

20. Find the sum of the infinite series if and a_1 48 $r=\frac{1}{2}$

A 24 **B** 48

C 96 **D** 192

$r=\frac{1}{2}$, $a_1=48$, $r=\frac{1}{2}$ since $|r|<1$,

then the GS_r is a converge series.

$$S=\frac{a_1}{1-r}$$

$$=\frac{48}{1-\frac{1}{2}}$$

$$=\frac{48}{\frac{1}{2}}$$

$$=96$$

➤ **C**

21. Find the sum $\sum_{k=1}^{\infty}\frac{5}{3}(\frac{3}{5})^k$

A $\frac{5}{2}$ **B** $\frac{5}{3}$

C $\frac{2}{5}$ **D** $\frac{3}{5}$

$$r=\frac{3}{5}$$

$$a_1=\left(\frac{5}{3}\right)\left(\frac{3}{5}\right)^1$$

$$=\frac{5}{3}.\frac{3}{5}$$

$$a_1=1$$

$$S=\frac{1}{1-\frac{3}{5}}$$

$$=\frac{1}{\frac{2}{5}}\rightarrow\frac{5}{2}$$

➤ **A**