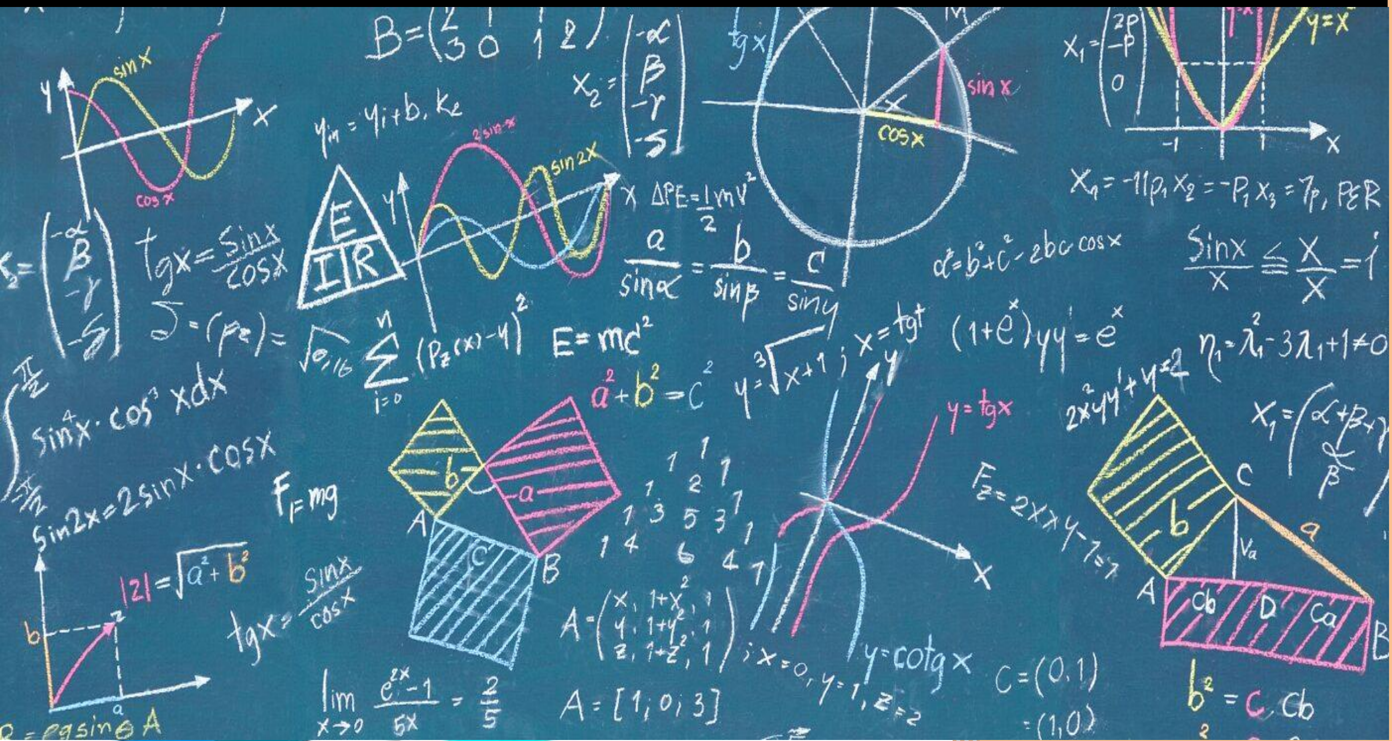
1444 H

ALGEBRA



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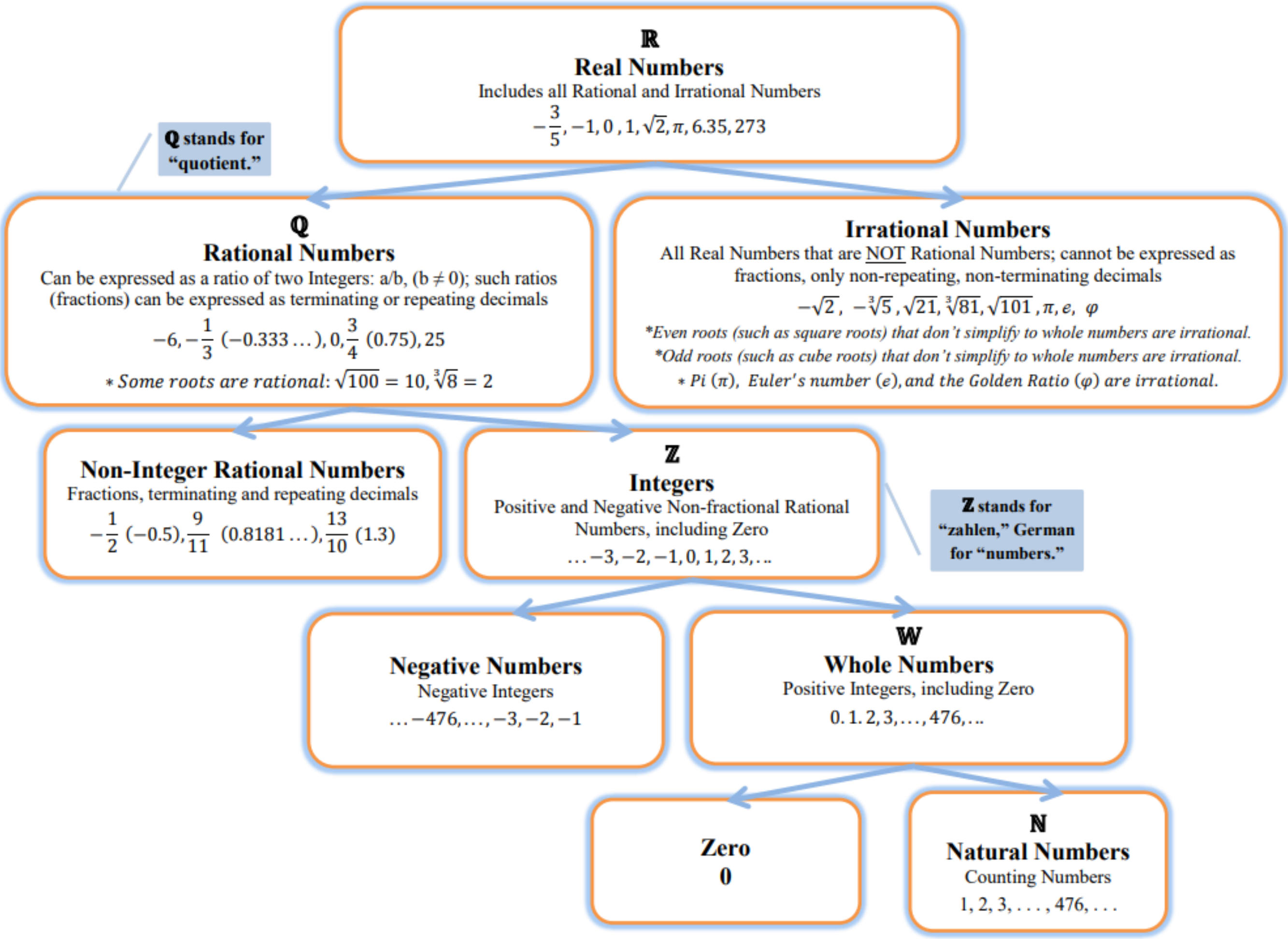
Set of Real Numbers

The set of real numbers consist of different categories, such as natural and whole numbers, integers, rational and irrational numbers. In the chart below, all these set of numbers are explained with examples.



1. Which of the following sets of numbers does number (–11) not belong to?

- **A** Z integers
- **B** Q Rational
- **C** R
- **D** W

(– 11) is negative number , the set of whole numbers does not include negative numbers.

⋙ D

2. What is the full classification of the number 120 ?

- **A** Irrational,real
- **B** Whole,Negative
- **C** Natural, Whole, Real
- **D** Irrational,

Number 120 is Real,Whole and Natural

⋙ C

3. Which number is different than the other numbers?

- **A** $\sqrt{21}$
- **B** $\sqrt{1000}$
- **C** $\sqrt{27}$
- **D** $\sqrt{0.25}$

The number $\sqrt{0.25} = 0.5$ is a rational number.

All other numbers are not perfect square,So they are irrational numbers.

⋙ D

4. What number of the following is irrational number?

- **A** $\sqrt{11}$
- **B** 3.14
- **C** $\sqrt[3]{-27}$
- **D** $0.\overline{25}$

The number $\sqrt{11}$ is not a perfect square So it is an irrational number.

⋙ A

Real Numbers Properties

Table of Properties

Let a, b, and c be real numbers, variables, or algebraic expressions.
(These are properties you need to know.)

	Property	Example
1.	Commutative Property of Addition $a + b = b + a$	$2 + 3 = 3 + 2$
2.	Commutative Property of Multiplication $a \cdot b = b \cdot a$	$2 \cdot (3) = 3 \cdot (2)$
3.	Associative Property of Addition $a + (b + c) = (a + b) + c$	$2 + (3 + 4) = (2 + 3) + 4$
4.	Associative Property of Multiplication $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$
5.	Distributive Property $a \cdot (b + c) = a \cdot b + a \cdot c$	$2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$
6.	Additive Identity Property $a + 0 = a$	$3 + 0 = 3$
7.	Multiplicative Identity Property $a \cdot 1 = a$	$3 \cdot 1 = 3$
8.	Additive Inverse Property $a + (-a) = 0$	$3 + (-3) = 0$
9.	Multiplicative Inverse Property $a \cdot \left(\frac{1}{a}\right) = 1$ Note: a cannot $= 0$	$3 \cdot \left(\frac{1}{3}\right) = 1$
10.	Zero Property $a \cdot 0 = 0$	$5 \cdot 0 = 0$

5. **What is the property used in the following expression** $-3x^2 + 4y = 4y - 3x^2$ **?**

- **A** Commutative Property
- **B** Distributive Property
- **C** Associative Property
- **D** Closure Property

Commutative Property:
$a + b = b + a$ ⇒ A

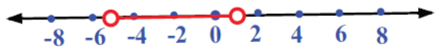
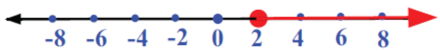
6. **What is the property used in the following expression** $-3\left(x^2 + 4y\right) = -3x^2 - 12y$ **?**

- **A** Commutative Property
- **B** Distributive Property
- **C** Associative Property
- **D** Closure Property

Distributive Property:
$a \cdot (b + c) = a \cdot b + a \cdot c$

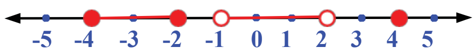
⇒ B

Interval Notation

Interval Type	Description	Inequality and interval notation	Graph
Closed interval	Includes its endpoints	$-5 \le x \le 1$	
	Use closed brackets [ ]	$[-5,1]$	
Open interval	Does not include its endpoints	$-5 < x < 1$	
	Use parentheses ( )	$(-5,1)$	
half–open and half–closed	Includes only one endpoint	$-5 \le x < 1$	
	Use combination of brackets and parenthesis	$[-5,1)$	
Intervals include infinity	Endpoints are $-\infty$, or ∞	$x \ge 2,\ [2,\infty)$	
		$x > 2,\ (2,\infty)$	
	Use combination of brackets and parenthesis as needed	$x \le 2,\ (-\infty, 2]$	
		$x < 2,\ (-\infty, 2)$	

- The set of all real number is represented by the interval notation: $(-\infty, \infty)$.
- You can also use interval notation together with the set union operator to write subsets of the number line made up of more than one interval:

$[-4,-2] \cup (-1,2) \cup \{4\}$



7. Find the interval that represents the inequality $-5 < x \le -2$?

A $[-5,-2]$ **B** $(-5,-2]$

C $[-5,-2)$ **D** $(-5,-2)$

- 5 is not included so we use () and -2 is included therefore we use[ ] **B**

8. Ahmed's expense in riyals per day can be represented by the following inequality $61 \le x < 362$

What is the largest value of his daily expenses?

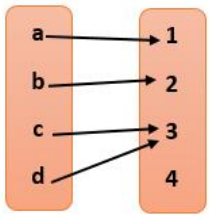
A SAR 61 **B** SAR 60

C SAR 362 **D** SAR 361

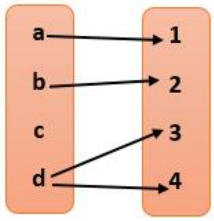
The inequality specifies that the greatest value is 362 but does not equal it, So the answer is one riyal less than 362 **D**

Functions

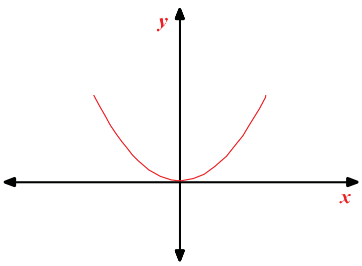
A function is a binary relation between two sets that associates each element of the domain (first set, *x*) to exactly one element of the range (second set, *y*).



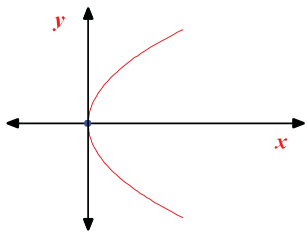
Function



Not Function



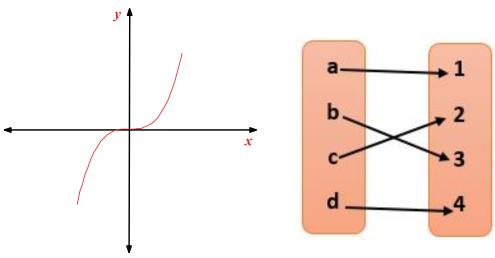
Function



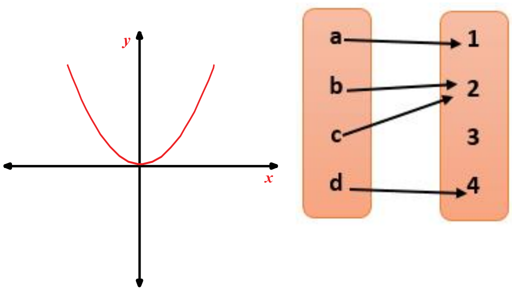
Not Function

One–to–One Functions

A function f is one–to–one if no two elements in the domain of f correspond to the same element in the range of f. In other words, each x in the domain has exactly one Value in the range, and no y in the range is the image of more than one x in the domain.

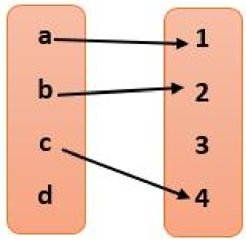


One–to–One Function



Not One–to–One Function

9. **Find the domain of the function**



A $\{a, b, c, d\}$ **B** $\{1, 2, 3\}$

C $\{1, 2, 3, 4\}$ **D** $\{a, b, c\}$

The domain is the elements of the first set that associated to elements in the second set **D**

10. **Find the domain of the function**

$\{(2, 4), (3, 5), (4, 6)\}$

A $\{1, 2, 3, 4, 5, 6\}$ **B** $\{2, 3, 4\}$

C $\{4, 5, 6\}$ **D** $\{1, 3, 5\}$

The domain is the first elements of the ordered pairs **B**

Equations

The equation $y = 3x^2 + 4x$, then y is called the dependent variable and x is the independent variable. Which means that the value of y depends on the value of x is. If we are considering functions, then the value of y could be represented by a function of x value ,So $y = 3x^2 + 4x$ can be written as $f(x) = 3x^2 + 4x$ where x is the input value and $f(x)$is the output value.

Example

$$f(x) = 4x^2 + 3x$$

A Find $f(2)$ **B** Find $f(-a)$

$f(x) = 4x^2 + 3x$

$f(2) = 4(2)^2 + 3(2)$

= 16 + 6

= 22

$f(x) = 4x^2 + 3x$

$f(2) = 4(-a)^2 + 3(-a)$

$f(2) = 4a^2 - 3a$

Example

If $f(x)=\begin{cases} x^2+4x & 1<x<3 \\ x+5 & x\geq 3 \end{cases}$

A Find $f(2)$

Because 2 is between 1 and 3, then we substitute 2 in the first equation that corresponds the interval $1<x<3$

$$f(x)=x^2+4x$$
$$f(2)=(2)^2+4(2)$$
$$=12$$

B Find $f(3)$

Since 3 ≥ 3, then we will substitute 3 in the second equation that corresponds the interval $X\geq 3$

$$f(x)=x+5$$
$$=(3)+5$$
$$=8$$

C Find $f(0)$

Since 0 < 1, then it does not in the domain of $f(x)$ therefore $f(0)$ is undefined.

11. If $f(x)=2x^2+4$, then find $f(3)$

A 11 **B** 16

C 10 **D** 22

$$f(x)=2x^2+4$$
$$f(3)=2(3)^2+4$$
$$=2\times 9+4=22$$

D

12. If $f(x)=2x^2+4$, then find $f(x-1)$

A x^2-2x+5 **B** $2x^2-4x+6$

C $2x^2+4x+6$ **D** x^2+2x+5

$$f(x)=2x^2+4$$
$$f(x-1)=2(x-1)^2+4$$
$$=2(x^2-2x+1)+4$$
$$=2x^2-4x+2+4$$
$$=2x^2-4x+6$$

B

13. If $f(x)=|2x-4|$, then find $f(-1)$

A -6 **B** 2

C -2 **D** 6

$$f(x)=|2x-4|$$
$$f(-1)=|2(-1)-4|$$
$$=|-2-4|$$
$$=|-6|=6$$

D

14. If $f(x)=\begin{cases} 4x+2 \ldots\ldots\ldots x\leq 1 \\ x^2+4x & 1<x<3 \\ x+5 & x\geq 3 \end{cases}$, then find $f(0)$

A 2 **B** 0

C 5 **D** Undefined

Since 0 ≤ 1, then we will substitute 0 in the first equation that corresponds the interval $X\leq 1$

$$f(x)=4x+2$$
$$=4(0)+2$$
$$=2$$

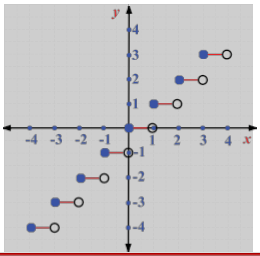
A

Greatest Integer Function

The symbol $\lfloor x \rfloor$ stands for the integer number less than or equal to X.

Example

Evaluate



A $\lfloor 2.9 \rfloor$ **B** $\lfloor -2.9 \rfloor$ **C** $\lfloor -4 \rfloor$

Solution
$\lfloor 2.9 \rfloor = 2$

Solution
$\lfloor -2.9 \rfloor = -3$

Solution
$\lfloor -4 \rfloor = -4$

1.15 Find the range of $f(x) = \lfloor x \rfloor + 3$

A R **B** Z

C $[3, \infty)$ **D** $(-\infty, 3]$

The range of the step function is Z >> **B**

1.16 Find the domain of $f(x) = \lfloor x \rfloor + 3$

A R **B** Z

C $[3, \infty)$ **D** $(-\infty, 3]$

The domain of the step function is R >> **A**

Monomials

A monomial is an expression in algebra that contains one term. Monomials include numbers (like 3), variables (like x) or a combination of them (like $98b$, mn , $5xy$ or $3x^2y^5$).

Degrees of a Monomial

Some monomials have an exponent. For example, $3y^2$ has an exponent of 2. It also has a degree of 2. In a monomial, you can add the exponents of the variables together to find the degree of a monomial function. The degree for a constant is always 0, and the degree for a variable that doesn't have an exponent listed is always 1

Example

Find the degree of the monomial $3a^2b^5c$
3 >> degree $= 0$, a^2 >> degree = 2, b^5 >> degree $= 5$, $= c^1$ >> degree $= 1$
The degree of the monomial is $0 + 2 + 5 + 1 = 8$

1.17 Which of the following monomials has a degree equal to the degree of the monomial $8a^2b^4c$?

A a^3bc^3 **B** $2a^2b^3c$

C $4a^3b^2c^3$ **D** $8abc^7$

The total degree of the monomial $8a^2b^4c$ is $0 + 2 + 4 + 1 = 7$, similarly we can find the total degree of the monomial a^3bc^3 is $3 + 1 + 3 = 7$ >> **A**

Power rules

$1^x = 1, \quad x \in R$

$x^1 = x, \quad x \in R$

$x^a \div x^b = \dfrac{x^a}{x^b} = x^{(a-b)}$

$x^a \times x^b = x^{(a+b)}$

$x^0 = 1, \quad x \neq 0$

$x^a = x^b \rightarrow a = b$

$(x^a)^b = x^{ab}$

Example

Simplify the following expressions

A $(-3x^3y^{-4})(2x^{-4}y^6)$ **B** $\dfrac{-30x^2y^{-1}}{6x^{-3}y^4}$

$$(-3x^3y^{-4})(2x^{-4}y^6) = (-3)\cdot\, 2x^{3+(-4)}y^{-4+6}$$
$$= -6x^{-1}y^2$$
$$= \frac{-6y^2}{x}$$

$$\frac{-30x^2y^{-1}}{6x^{-3}y^4} = \frac{-30}{6}x^{2-(-3)}y^{-1-4}$$
$$= -5x^5y^{-5}$$
$$= \frac{-5x^5}{y^5}$$

Polynomials

Polynomials are algebraic expressions that consist of variables and coefficients. We can perform arithmetic operations such as addition, subtraction, multiplication and also positive integer exponents for polynomial expressions but not division by variable. Polynomials are named either by its degree or by the number of its terms.

Polynomial	Degree	Examples
Constant or Zero Polynomial	0	6
Linear Polynomial	1	$3x+1$
Quadratic Polynomial	2	$4x^2+2x+5$
Cubic Polynomial	3	$6x^3-x^2+5x+1$
Quartic Polynomial	4	x^4+3x+7

Polynomial	Number of terms	Examples
Monomial	One term	$2x^3,\quad \frac{x^2}{3},\quad 5y,\quad 1$
Binomial	Two terms	$x^4+3x,\quad 3y+4,\quad 2x+7$
Trinomial	Three terms	$5x^3+2x+8$

1.18 Which of the following polynomials is classified as cubic polynomial?

A x^4+x^2+8 **B** 7^3+2x^2+8

C $5x^3+2x-x^4$ **D** $x^3+2x+8x^3$

The cubic polynomial degree should be 3, simplifying $x^3+2x+8x^3$ ∨ get $9x^3+2x$ of degree 3 ⇒ **D**

1.19 Find the leading coefficient of the polynomial $-5x^3+2x-8x^4$

A 5 **B** -5

C 8 **D** -8

The leading coefficient of any polynomial is the coefficient of term that has the greatest exponent $-8x^4 \rightarrow -8$ ⇒ **D**

Greatest Common Factor GCF & Least Common Multiple LCM

GCF & LCM of two numbers or more

To find the **GCF** & **LCM** of two numbers or more we can use the Ladder division method, where we keep dividing by common factors till no more common factors

Example

Find GCF and LCM of the following numbers

A GCF & LCM of 12 and 18

Solution

$$\begin{array}{c|cc} 2 & 12 & 18 \\ \hline 3 & 6 & 9 \\ \hline & 2 & 3 \end{array} \qquad \begin{array}{c|cc} 6 & 12 & 18 \\ \hline & 2 & 3 \end{array}$$

GCF: $2\times3=6$

LCM: $2\times3\times2\times3=36$

B GCF & LCM of 8, 12 and 16

Solution

$$\begin{array}{c|ccc} 4 & 8 & 12 & 16 \\ \hline & 2 & 3 & 4 \end{array}$$

GCF: $\rightarrow 4$

LCM: Choose the greatest two numbers 12 and 16

$$\begin{array}{c|cc} 4 & 12 & 16 \\ \hline & 3 & 4 \end{array}$$

$4\times3\times4=48$

Then if 48 divides the least number (8) then it will be the LCM otherwise multiply 48 by 2, 3, …

In our case $48\div8=6$ therefore LCM: $4\times3\times4=48$

GCF & LCM of two monomials or more

To find the greatest common factor **(GCF)** of a set of monomials, find the **GCF** of the constants and the **GCF** of each of the variables. After finding these, multiply them all together to get the **GCF** of the set of monomials.

To find the least common multiple **(LCM)** of a set of monomials, find the **LCM** of the constants and the **LCM** of each of the variables. After finding these, multiply them all together to get the **LCM** of the set of monomials.

Example

Find GCF and LCM of the following polynomials

A $x^4y^4, \quad x^3y \quad ,x^2y^5$

Solution

GCF: Choose the minimum exponent of any variable that all terms have in common $\rightarrow x^2y$

LCM: Choose the maximum exponents of all variables $\rightarrow x^4y^5$

B $xy^4z, \quad x^3y \quad ,w^2y^5$

Solution

GCF: Choose the minimum exponent of any variable that all terms have in common $\rightarrow y$

LCM: Choose the maximum exponents of all variables $\rightarrow w^2x^3y^5z$

Factoring and multiplying

$$(a-b)=-(b-a)$$

Example

$$(x-5)=-(5-x)$$

Factoring by GCF

You are advised to use GCF factorization before solving any question to simplify it

Step 1: Determine the greatest common factor GCF of the given terms.
Step 2: Factor out (divide) the greatest common factor from each term.

Example

Factor the following polynomials using GCF

A $16x^2-12x$

Solution

Step 1: GCF is $4x$

Step 2: Divide by GCF

$$4x\left(\frac{16x^2}{4x}-\frac{12x}{4x}\right)=4x(4x-3)$$

B $12x^5-18x^3-3x^2$

Solution

Step 1: GCF is $3x^2$

Step 2: Divide by GCF

$$3x^2\left(\frac{12x^5}{3x^2}-\frac{18x^3}{3x^2}-\frac{3x^2}{3x^2}\right)=3x^2\left(4x^3-6x-1\right)$$

C $15x^3y^2+10x^2y^4$

Solution

Step 1: GCF is $5x^2y^2$

$$5x^2y^2\left(\frac{15x^3y^2}{5x^2y^2}+\frac{10x^2y^4}{5x^2y^2}\right)=5x^2y^2\left(3x+2y^2\right)$$

Factoring by Grouping

If we have a polynomial that has four terms we may apply factoring by grouping $ax+bx+ay+by=x(a+b)+y(a+b)$

$$=(x+y)(a+b)$$

Example

Factor the polynomial $x^3-5x^2+3x-15$

Solution

Step 1: Create smaller groups within the problem, usually done by grouping the first two terms together and the last two terms together. $x^3-5x^2+3x-15$

Step 2: Factor out the GCF from each of the two groups. $x^2(x-5)+3(x-5)$

Step 3: The one thing that the two groups have in common is $(x-5)$, so you can factor out $(x-5)$ leaving the following: $(x^2+3)(x-5)$

Factoring ax^2+bx+c

Multiply the coefficient of the leading term a by the constant term c. List the factors of this product $(a \cdot c)$ to find the pair of factors, f_1 and f_2, that sums to b, the coefficient of the middle term.

Example

Factor the following polynomials

A $6x^2+x-2$

Solution

$a=6,\ b=1,\ c=-2$

Step 1: $ac=6\times(-2)=-12$

Step 2: Factors (12): $(1, 12), (2, 6), (3, 4)$

Step 3: Use the factors to find the value of $b\to 1$, by addition or subtraction $4+(-3)=1$

Step 4: Rewrite the equation in four terms then factoring by grouping

$$\begin{aligned}6x^2+x-2 &= 6x^2+4x-3x-2\\ &=(6x^2+4x)-(3x+2)\\ &=2x(3x+2)-(3x+2)\\ &=(2x-1)(3x+2)\end{aligned}$$

B $x^2+7x+12$

Solution

$a=1,\ b=7,\ c=12$

Step 1: $ac=1\times 12=12$

Step 2: Factors (12): $(1, 12), (2, 6), (3, 4)$

Step 3: Use the factors to find the value of $b\to 7$, by addition or subtraction $4+3=7$

Step 4: Since a $=1$, we can factor directly by using the pre chosen factors: 4 & 3

$$x^2+7x+12=(x+3)(x+4)$$

C $x^2-4x-12$

Solution

$a=1,\ b=-4,\ c=-12$

Step 1: $ac=1\times 12=12$

Step 2: Factors (12): $(1, 12), (2, 6), (3, 4)$

Step 3: Use the factors to find the value of $b\to -4$, by addition or subtraction $-6+2=-4$

Step 4: Since a $=1$, we can factor directly by using the pre chosen factors: -6 & 2

$$x^2-4x-12=(x-6)(x+2)$$

Complex numbers

A complex number is a number that can be expressed in the form $a+bi$, where a is the real unit and bi is the imaginary unit, the imaginary number is $i=\sqrt{-1}$

$$i^1 = i$$
$$i^2 = \sqrt{-1}\times\sqrt{-1} = -1$$
$$i^3 = i^1\times i^2 = i^1\times(-1) = -i$$
$$i^4 = i^2\times i^2 = -1\times(-1) = 1$$
$$i^5 = i^4\times i^1 = 1\times i = i$$

The pattern of i^n is repeated for all multiples of 4, therefore to simplify i^n

Step 1: Find $n\div 4$

Step 2: Check if the remainder is

Remainder	Value
0	1
1	i
2	– 1
3	$-i$

Example: Simplify

A i^{16}

Solution

Step 1: Find $16\div 4 = 4 \quad R=0$

Step 2: Check the remainder

Remainder	Value
0	1

B i^{39}

Solution

Step 1: Find $39\div 4 = 9 \quad R=3$

Step 2: Check the remainder

Remainder	Value
3	$-i$

1.20 Find the value of $\sqrt{-25}$

A -5 **B** $-5i$

C 5 **D** $5i$

$$\sqrt{-25} = \sqrt{-1\times 25}$$
$$= \sqrt{-1}\times\sqrt{25}$$
$$= 5i$$

➢ D

21. Evaluate $2i\times 7i$

A 14 **B** -14

C $14i$ **D** $-14i$

$$2i\times 7i = (2\times 7)\times(i\times i)$$
$$= 14\times i^2$$
$$= 14\times(-1)$$
$$= -14$$

➢ B

1.22 Evaluate $(1-i)^6$

A 8 **B** -8

C $8i$ **D** $-8i$

$$(1-i)^6 = [(1-i)^2]^3$$
$$= [1-2\times 1\times i + i^2]^3$$
$$= [1-2i+(-1)]^3$$
$$= [-2i]^3$$
$$= (-2)^3(i)^3$$

$$= -8(-i) = 8i$$

➢ C

Operations on Complex Numbers

To add or subtract complex numbers we apply the required operation to the corresponding parts
for example: $(3-8i)+(2+5i)=(3+2)+(-8+5)i=5-3i$

To multiply two complex numbers we use FOIL or distributive in the same way as we did in multiplying polynomials $(a+b)(c+d)=ac+ad+bc+bd$

Example

Multiply the following two complex numbers $(2+3i)(1-2i)$

Solution

$$\begin{aligned}(2+3i)(1-2i) &= 2\cdot1+2\cdot-2i+3i\cdot1+3i\cdot-2i\\ &= 2\quad -4i\quad +\quad 3i\quad -\quad 6i^2\\ &= 2-i-6\cdot-1\\ &= 2-i+6\\ &= 8-i\end{aligned}$$

23. **Find the product of** $(2+5i)(1+2i)$

A $-8+9i$ **B** $8+9i$

C $-8-9i$ **D** $8-9i$

$$\begin{aligned}(2+5i)(1+2i) &= 2\cdot1+2\cdot2i+5i\cdot1+5i\cdot2i\\ &= 2\quad +\quad 4i\quad +\quad 5i\quad +\quad 10i^2\\ &= 2\quad +\quad 9i\quad -\quad 10\\ &= -8\quad +\quad 9i\end{aligned}$$

A

24. **Simplify** $(3+5i)-(-2+i)$

A $5+6i$ **B** $5+4i$

C $5-6i$ **D** $1+4i$

$$\begin{aligned}(3+5i)-(-2+i) &= 3-(-2)+5i-i\\ &= 5+4i\end{aligned}$$

B

Conjugate of complex number

The conjugate of complex is another complex number that has the same real as the original complex number and the imaginary part has the same magnitude but opposite sign. The product of a complex number and its complex conjugate is a real number. $(a+bi)(a-bi)=a^2+b^2$

To divide by a complex number or to simplify a fraction that has a complex number denominator we multiply by $\frac{conjugate}{conjugate}$ of the divisor or the denominator.

Example

Simplify $\frac{2+i}{3-2i}$

Solution

$$\begin{aligned}\frac{2+i}{3-2i} &= \frac{2+i}{3-2i}\cdot\frac{3+2i}{3+2i}\\ &= \frac{6+4i+3i+2i^2}{3^2+2^2}\\ &= \frac{4+7i}{13}\\ &= \frac{4}{13}+\frac{7}{13}i\end{aligned}$$

25. **Simplify** $\frac{i-1}{2i}$

A $\frac{1}{2}+\frac{1}{2}i$ **B** $\frac{1}{2}-\frac{1}{2}i$

C $-\frac{1}{2}i$ **D** $\frac{-1}{2}-\frac{1}{2}i$

$$\begin{aligned}\frac{i-1}{2i} &= \frac{i-1}{2i}\cdot\frac{-2i}{-2i}\\ &= \frac{i-1}{2i}\cdot\frac{i}{i}\\ &= \frac{i(i-1)}{2i^2}\\ &= \frac{i^2-i}{-2}\\ &= \frac{-1-i}{-2}\\ &= \frac{-1}{-2}+\frac{-i}{-2}\\ &= \frac{1}{2}+\frac{1}{2}i\end{aligned}$$

A

Two complex numbers are said to be equal if and only if their real parts are equal and their imaginary parts are also equal.

Example

Find X and y that makes the equation true $x+6i=3-2yi$

Real part **imaginary part**

$$6=-2y$$

$$x=3 \qquad y=\frac{6}{-2}$$

$$=-3$$

Quadratic Formula and Discriminant

To solve any quadratic equation of the form ax^2+bx+c you can use the quadratic formul $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

You can use the discriminant to determine the number of real roots (solutions) of a quadratic equation.

The discriminant is the radicand b^2-4ac in the quadratic formula.

Equation	b² - 4ac	Discriminant	Roots
x^2+5x+6	$25-4\cdot1\cdot6=1$	>0	2 real solutions, roots
x^2+6x+9	$36-4\cdot1\cdot9=0$	$=0$	1 real solutions , roots of multiplicity 2
$2x^2-3x+3$	$9-4\cdot2\cdot3=-15$	<0	2 complex solutions , roots

26. Find the discriminant of $x^2-2x=0$

A 4 **B** 0

C -8 **D** -4

$$x^2+3x=(1)x^2+(-2)x+0=0$$

Discriminant

$$=b^2-4ac$$

$$a=1,\quad b=-2,\quad c=0$$

$$=(-2)^2-4\cdot1\cdot0$$
$$=4$$

A

27. How many roots dose the following equation have?

$$-x+4x^2+2=0$$

A 2 different real roots

B 2 complex roots

C 1 real and 1 complex root

D 1 real root of multiplicity 2

$$-x+4x^2+2=4x^2-x+2 \quad \textbf{Standard form}$$

$$=(4)x^2+(-1)x+2$$

$$a=4 \qquad b=-1 \qquad c=2$$

$$\text{discriminat} \quad =b^2-4ac$$

$$=(-1)^2-4\cdot4\cdot2$$
$$=1-32$$
$$=-31<0 \ \text{negative}$$

B

28. Solve the equation $x^2 - 4x + 5 = 0$

- **A** $x = -2 \pm i$
- **B** $x = 2 \pm 2i$
- **C** $x = 2 \pm i$
- **D** $x = 1 \pm i$

$$x^2 - 4x + 5 = (1)x^2 + (-4)x + 5$$
$$a = 1 \quad b = -4 \quad c = 5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$
$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$
$$= \frac{4 \pm \sqrt{-4}}{2}$$
$$= \frac{4 \pm 2i}{2}$$
$$= 2 \pm i$$

C

29. Which equation has a real root of multiplicity 2

- **A** $x^2 = 18$
- **B** $x^2 - 2x + 6 = 0$
- **C** $x^2 - 3x - 4 = 0$
- **D** $x^2 + 10x = -25$

Method (1)

The perfect square equation has 1 root of multiplicity 2 in the perfect square $c = \left(\frac{b}{2}\right)^2$

The equation in option 'D' is a perfect square

$$x^2 + 10x = -25 \longrightarrow x^2 + 10x + 25 = 0$$
$$a = 1 \quad , \quad b = 10 \quad , \quad c = 25$$
$$c = \left(\frac{b}{2}\right)^2 \longrightarrow 25 = \left(\frac{10}{2}\right)^2 \longrightarrow True$$

Method (2)

Check all the discriminant of the equations

$$b^2 - 4ac = (10)^2 - 4 \cdot 1 \cdot 25$$
$$= 100 - 100 = 0$$

If the discriminant = 0 then the equation has 1 real root of multiplicity 2

D

Special cases

Case	Multiply $\longrightarrow$		$\longleftarrow$ Factor
Perfect square	$(\ \pm\ )^2$	=	$a^2 + 2ab + b^2$
Example	$(x+5)^2$	=	$x^2 + 2 \cdot 5 \cdot x + 5^2$ $x^2 + 10x + 25$
Difference of squares	$(\ -\ )(a+b)$	=	$a^2 - b^2$
Example	$(3x+7)(3x-7)$	=	$9x^2 - 49$

30. Identify the prime polynomial

- **A** $4x^2 - 16y^2$
- **B** $4x + 8$
- **C** $4x^2 - 10x^2$
- **D** $5x - 3$

The prime polynomial is a polynomial that we can't factorize in any method of the previous methods.

The equation $5x - 3$ can't be factorized any more.

D

31. If $x^2 - y^2 = 24$ **and** $x + y = 6$, **then find** $x - y$

- **A** 6
- **B** 4
- **C** 8
- **D** 3

$$x^2 - y^2 = (x-y)(x+y) \quad \text{Difference Squares}$$
$$24 = (x-y) \cdot 6$$
$$\frac{24}{6} = (x-y) \quad \text{Substitute}$$
$$4 = (x-y)$$

B

Adding and Subtracting Polynomials

To add or subtract Polynomials we start by combining like terms then we get rid of parentheses and use the distributive property as needed.

Finally, we check again for like terms.

Example

Simplify $(3x^2+3x+14+3x)+x(x-1)$

Solution

$$(3x^2+3x+14+3x)+x(x-1)$$
$$(3x^2+6x+14)+x(x)+x(-1)$$
$$3x^2+6x+14+x^2-x$$
$$4x^2+5x+14$$

32. Simplify $(-4x^2+2x+1)-2(x^2-5x+1)$

A $-6x^2+12x$ **B** $-6x^2+12x-1$

C $6x^2+12x-1$ **D** $12x-1$

$$(-4x^2+2x+1)-2(x^2-5x+1)=-4x^2+2x+1-2x^2+10x-2$$
$$=-6x^2+12x-1$$

B

33. Simplify $\frac{1}{2}x^2(6x^2+8x-4)$

A $3x^4+4x^3-2x^2$ **B** $3x^3+4x^2-2x$

C $3x^2+4x-2$ **D** x^4+4x^3-2x

$$\frac{1}{2}x^2(6x^2+8x-4)=\frac{1}{2}x^2(6x^2)+\frac{1}{2}x^2(8x)+\frac{1}{2}x^2(-4)$$
$$=\frac{6}{2}x^{2+2}+\frac{8}{2}x^{2+1}+\frac{-4}{2}x^2$$
$$=3x^4+4x^3-2x^2$$

A

Rational Expressions

A rational expression is simply a quotient of two polynomials. It is a fraction whose numerator and denominator are polynomials. Since dividing by zero is undefined, then the denominator must not be zero, otherwise the rational expression will be undefined.

Therefore the zero of the denominator are excluded from the domain of the rational expression.

Example

Find the values that make the following rational expression undefined $\frac{x+5}{(x-2)(x+3)}$

Solution

let the denominator = 0

$$(x-2)(x+3)=0$$
$$x-2=0 \quad or \quad x+3=0$$
$$x=2 \qquad x=-3$$

So the expression is undefined when $x=2$ or $x=-3$

34. The rational expression $\frac{x-4}{x^2-16}$ **is undefined at ...**

A $x=16$ **B** $x=4$

C $x=\pm4$ **D** $x=\pm16$

$$x^2-16=0$$
$$(x-4)(x+4)=0$$
$$x-4=0 \quad or \quad x+4=0$$
$$x=4 \qquad x=-4$$
$$x=\pm4$$

C

35. If $f(x)=3x+9$ **and** $g(x)=x+4$, **then find the domain of** $\left(\frac{f}{g}\right)x\left(\frac{g}{f}\right)$

A all real numbers **B** $\{x \mid x\neq-3\}$

C $\{x \mid x\neq-3, x\neq-4\}$ **D** $\{x \mid x\neq3, x\neq4\}$

Since both $f(x)$ and $g(x)$ are denominators, so both can't be zero

$$f(x)\neq0$$
$$f(x)=3x+9\neq0\longrightarrow x\neq-3$$
$$g(x)\neq0$$
$$g(x)=x+4\neq0\longrightarrow x\neq-4$$
$$\{x \mid x\neq-3, x\neq-4\}$$

A

Multiplying Rational Expression

To multiply rational expressions

1- Completely factor all numerators and denominators.
2- Simplify all common factors.

Example

Multiply $\frac{x^5}{x-6} \cdot \frac{x^2-6x}{x^8}$

Solution

$$\frac{x^5}{(x-6)} \cdot \frac{x(x-6)}{x^8}$$ X as common factor

$$= \frac{x^5 \cdot x^1}{x^8}$$

$$= x^{5+1-8}$$ $x^a \cdot x^b = x^{a+b}$, $\frac{x^a}{x^b} = x^{a-b}$

$$= x^{-2}$$

$$= \frac{1}{x^2}$$

36. **Multiply** $\frac{x^2-y^2}{3y} \cdot \frac{9y^2}{x-y}$

A $3(x+y)$ **B** $3y(x+y)$

C $3y(x-y)$ **D** $9x^2y^4 - 9y^4$

$$\frac{x^2-y^2}{3y} \cdot \frac{9y^2}{x-y} = \frac{(x-y)(x+y)}{3y} \cdot \frac{9y^2}{(x-y)}$$ **Difference of squares**

$$= \frac{9y^2(x+y)}{3y}$$

$$= \frac{9}{3} y^{2-1}(x+y)$$

$$= 3y(x+y)$$

B

Dividing Rational Expressions

Rational expressions are divided in the same way just like dividing fractions. To divide two fractions, we multiply the first fraction by the reciprocal of the second fraction.

Example

Simplify $\frac{5a}{2b} \div \frac{10a}{4b}$

Solution

$$\frac{5a}{2b} \div \frac{10a}{4b} = \frac{5a}{2b} \times \frac{4b}{10a}$$ **Flip and multiply**

$$= \frac{5}{10} \cdot \frac{4}{2} \cdot \frac{a}{a} \cdot \frac{b}{b}$$ **Commutative property**

$$= \frac{1}{2} \cdot \frac{2}{1} \cdot 1 \cdot 1 = 1$$

Example

Find quotient of $\frac{x^4+2x^3-2x^2-3x+2}{x+2}$

Solution

$$\frac{x^4+2x^3-2x^2-3x+2}{x+2} = \frac{x^3(x+2)-(2x^2+3x-2)}{x+2}$$

$$= \frac{x^3(x+2)-(2x-1)(x+2)}{(x+2)}$$

$$= x^3 - 2x + 1$$

37. **Simplify** $\frac{x(x^2+3x-18)}{(x+3)(x-4)} \div \frac{x(x+6)}{x+3}$

A $\frac{x-3}{x-4}$ **B** $\frac{x+3}{x+4}$

C $\frac{x+3}{x-4}$ **D** $\frac{x-4}{x+3}$

$$\frac{x(x^2+3x-18)}{(x+3)(x-4)} \div \frac{x(x+6)}{x+3} = \frac{x(x+6)(x-3)}{(x+3)(x-4)} X \frac{x+3}{x(x+6)}$$

simplify then we get $= \frac{x-3}{x-4}$

A

38. **Simplify** $(x^2+x-20)(x-4)^{-1}$

A $x+5$ **B** $x-5$

C $x-4$ **D** $x+4$

$$(x^2+x-20)(x-4)^{-1} = \frac{x^2+x-20}{x-4}$$

$$= \frac{(x-4)(x+5)}{(x-4)}$$

$$= x+5$$

A

39. Find the width of the rectangle if its length is $3x+4$ and its area is $3x^2-2x-8$

A $x+2$ **B** $x-2$

C $3x+4$ **D** $3x-4$

$$A = wl \longrightarrow w = \frac{A}{l} = \frac{3x^2-2x-8}{3x+4}$$
$$= \frac{(3x+4)(x-2)}{(3x+4)}$$
$$= x-2$$

➢ **B**

Remainder Theorem

It states that the remainder of the division of a polynomial $f(x)$ by a linear polynomial $x-r$ is equal to $f(r)$

Example

Find the remainder of dividing $f(x) = x^3+2x-1$ by $x-1$

Since the divisor is $x-1$ we will find $f(1) \longrightarrow f(1) = (1)^3 + 2(1) - 1$

$$= 1+2-1$$
$$= 2$$

40. Find the linear equation that if $f(x) = x^2-4x+5$ is divided by, then the remainder would be 2

A $x+3$ **B** $x-3$ **C** $x+2$ **D** $x-2$

Method 1: Using Remainder Theorem

Step 1: By trial and error we find $f(-3), f(3), f(-2)$ and $f(2)$ and choose the option that leads to 2.

$$f(3) = (3)^2 - 4(3) + 5$$
$$= 9-12+3$$
$$= 2$$

Step 2: $x = 3$

$x-3=0$

Method 2: Use synthetic division to divide the function by all options but don't forgot to flip the sign

Divide by option B $\rightarrow$ $x-3$

$$\begin{array}{r|rrr} 3 & 1 & -4 & 5 \\ & & 3 & -3 \\ \hline & 1 & -1 & \boxed{2} \end{array}$$

$x = 3$

$x-3=0$

$\rightarrow f(x) = x-3$

➢ **B**

41. Find k such that if we divide $f(x) = x^3 - kx + 4$ by $x+2$ the remainder would be 2

A -3 **B** 3 **C** 2 **D** -2

$(x-r) \rightarrow x+2 = x-(-2) \rightarrow r = -2$

By remainder theorem $f(-2) = 2$

$$f(x) = x^3 - kx + 4$$
$$f(-2) = (-2)^3 - k(-2) + 4$$
$$2 = -8+2k+4$$
$$2 = -4+2k$$
$$6 = 2k$$
$$k = 3$$

➢ **B**

Factors of polynomials

If $f(r)=0$, that means the remainder is 0 , and then $(x-r)$ is a factor of the polynomial

42. **Determine the factor of** $f(x)=x^3-2x^2-x+2$

A $x-2$ **B** $x+2$

C $x-1$ **D** $x+1$

By trial and error we find

$f(2), f(-2), f(1)$ and $f(-1)$

and choose the option that leads to zero

then the $x-r$ **will be the factor of** $f(x)$

$$f(x)=x^3-2x^2-x+2$$
$$f(2)=2^3-2\cdot 2^2-2+2$$
$$=8-8-2+2$$
$$=0$$

A

43. **one of the follows factors is not a factor of the polynomial** $f(x)=x^4+2x^3-x^2-2x$

A x **B** $(x+1)$

C $(x+2)$ **D** $(x+3)$

If $(x-r)$ **is a factor of** $f(x)$

then $f(r)=0$ **And vice versa.**

Therefore we should find $f(r)\neq 0$

$$x+3\to x-(-3)\to r=-3$$
$$f(-3)=(-3)^4+2(-3)^3-(-3)^2-2(-3)$$
$$=81+2(-27)-9+6\neq 0$$

D

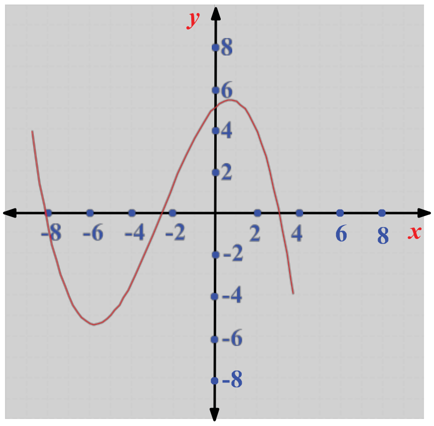
Roots of Polynomial equations

The roots (also called zeros or solutions) of a polynomial $p(x)$ are the values of x for which $p(x)$ is equal to zero.

To find the zero algebraically we let $p(x)=0$. The roots on the graph are the intersection points of the curve of and the $p(x)$ *x-axis*.

Example

How many roots dose $f(x)$ **have**



Solution:

Number of the roots is the number of x intercepts the curve intercepts the $x-\text{axis}$ **3 times therefore** $f(x)$ **has 3 roots.**

Note: **the number of roots of a polynomial is the same as its degree.**
For example $f(x)=2x^5+3x^3+x^2+1$
has five roots.

44. **Find a zero of the polynomial**

$f(x)=x^2-5x-6$

A 3 **B** 2

C 6 **D** -6

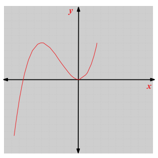
$$f(x)=0$$
$$x^2-5x-6=0$$
$$(x+1)(x-6)=0$$
$$x+1=0\to x=-1$$
$$x-6=0\to x=6$$

Only $x=6$ **is an option**

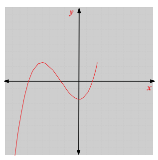
C

45. **which graph shows that the polynomial has four real roots**

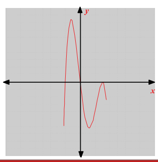
A



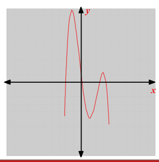
B



C



D



The graph of $f(x)$

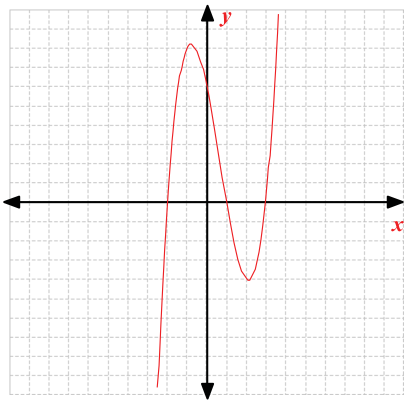
has four X**-intercepts**

D

46. One of the following factors is not a factor of $f(x)$

A $x-1$ **B** $x+2$

C $x-3$ **D** $x+1$



The roots are x-intercepts

$-2,1,3 \rightarrow$ the factors are:

$x+2,\quad x-1,\quad x-3$ respectively

therefore $x+1$ is not a factor ⇒ D

Complex Roots

For any polynomial, if a complex number is a root of the polynomial then its conjugate is also a root of the polynomial.

47. Find the least degree of the polynomial that has the zeros: $7,(3-2i)$

A 1 **B** 2

C 3 **D** 4

Since the conjugate must also be a root of the polynomial, therefore the polynomial has 3 root and it degree is 3 ⇒ C

48. What is the maximum number of complex root that polynomial of the fifth degree may have?

A 5 **B** 2

C 3 **D** 4

Since complex root occurs in pairs, then maximum number of complex roots is 4 ⇒ D

Composite Function

$(f\circ g)(x)$ is a composite function of $f(x)$ and $g(x)$

The composite function is read as "f of g of x"

$(f\circ g)(x)=f\left[g(x)\right]$

The steps required to perform this operation is to start by the inner function $g(x)$ and substituting it in the outer function which is $f(x)$

Example

If $f(x)=4x^2, g(x)=5x,$

then find $(f\circ g)(2)$

Solution:

$$\begin{aligned}(f\circ g)(2)&=f[g(2)]\\&=f(5\cdot 2)\\&=4(10)^2 \qquad f(x)=4x^2\\&=4\cdot 100\\&=400\end{aligned}$$

Example

If $f(x)=4x^2, g(x)=3x+1,$

then find $(f\circ g)(x)$

Solution:

$$\begin{aligned}(f\circ g)(x)&=f\left[g(x)\right]\\&=f(3x+1)\\&=4(3x+1)^2 \qquad f(x)=4x^2\\&=4(9x^2+6x+1)\\&=36x^2+24x+4\end{aligned}$$

Example

If $f(x)=\{(8,2),(2,-1),(5,-3)\},\quad g(x)=\{(2,-3)(4,2)(5,8)\}$

then find $(f\circ g)(x)$

Solution:

$(f\circ g)(x)=f[g(x)]$

$g(2)=-3\rightarrow f(-3)$ undefined

$g(4)=2\rightarrow f(2)=-1$

$g(5)=8\rightarrow f(8)=2$

$\rightarrow(f\circ g)(x)=\{(4,-1),(5,2)\}$

49. If $f(x)=3x, (f\circ g)(x)=3x+3$

then find $g(x)$

A x **B** $x+3$

C $x+1$ **D** $3x$

$$(f\circ g)(x)=f[g(x)]=3x+3$$
$$=3(x+1) \text{ Common factor}$$
$$=3g(x) \qquad f(x)=3x$$
$$\rightarrow g(x)=x+1$$

C

50. If $f(x)=x^2, g(x)=\sqrt{x^2+16}$

then find $(f\circ g)(x)$

A $x+4$ **B** $x\pm 4$

C x^2+16 **D** $x^2\pm 16$

$$(f\circ g)(x)=f[g(x)]$$
$$=f\left(\sqrt{x^2+16}\right)$$
$$=\left(\sqrt{x^2+16}\right)^2$$
$$=x^2+16$$

C

51. If $f(x)=\sqrt{x^2+9},\ g(x)=\sqrt{x^2-9},\ (f\circ g)(x)=3$

then find x

A $\pm\sqrt{3}$ **B** 3

C ± 3 **D** $\sqrt{3}$

$$(f\circ g)(x)=f[g(x)]=f\left(\sqrt{x^2-9}\right)$$
$$3=\sqrt{\left(\sqrt{x^2-9}\right)^2+9}$$
$$3=\sqrt{x^2-9+9}$$
$$3=\sqrt{x^2}$$
$$3=\pm x$$
$$\pm 3=x$$

C

52. If $f(x)=x^2+1,\ g(x)=x-3,\ (f\circ g)(x)=3$

then find x such that $(f\circ g)(x)=(g\circ f)(x)$

A 0 **B** 1

C 2 **D** 3

$$(f\circ g)(x)=(g\circ f)(x)$$
$$f[g(x)]=g[f(x)]$$
$$f(x-3)=g(x^2+1)$$
$$(x-3)^2+1=x^2+1-3$$
$$x^2-6x+9+1=x^2-2$$
$$-6x+10=-2$$
$$-6x=-12$$
$$x=2$$

C

53. If $f(x)=x^2+3x$ and $g(x)=4k$

then find $(f\circ g)(x)$

A $8k^2+12k$ **B** $16k^2+12k$

C $4k^2+12k$ **D** $8k^2+7k$

$$(f\circ g)(x)=f[g(x)]=f(4k)$$
$$=(4k)^2+3(4k)$$
$$=16k^2+12k$$

B

Inverse Function

The inverse function of $f(x)$ is denoted by $f^{-1}(x)$. To find the inverse function of $f(x)$ we rewrite $f(x)$ as y, then swap y by x, and solve for y which is $f^{-1}(x)$.

Example

Find the inverse of $f(x)$ if $f(x)=4x-5$

Solution

Rewrite $f(x)$ as y

$$y=4x-5$$

$$x=4y-5 \qquad \text{swap } y \text{ by } x$$

$$-4y=-x-5 \qquad \text{solve for } y$$

$$y=\frac{-x}{-4}+\frac{-5}{-4} \qquad \div \text{ by } (-4)$$

$$f^{-1}(x)=\frac{x}{4}+\frac{5}{4}$$

Note:

Original Function	Inverses Function
$f(x)$	$f^{-1}(x)$
Domain	Domain
Range	Range

54. Find the invers function of $f(x)=\frac{x-4}{3}$

A $3x+4$ **B** $4x+3$

C $\frac{x-3}{4}$ **D** $3x-4$

$$f(x)=\frac{x-4}{3}$$

$$y=\frac{x-4}{3}$$

$$x=\frac{y-4}{3}$$

$$3x=y-4$$

$$y=3x+4$$

$$f(x)=3x+4$$

A

55. if $f(x)=(2x-1)(2x^{-1})$, **then find** $f^{-1}(x)$

A $4x-3$ **B** $\frac{2}{x-4}$

C $\frac{x-3}{4}$ **D** $\frac{-2}{x-4}$

$$f(x)=(2x-1)(2x^{-1})$$

$$=\frac{(2x-1)\cdot 2}{x}$$

$$f(x)=\frac{4x-2}{x}$$

$$y=\frac{4x-2}{x}$$

$$x=\frac{4y-2}{y}$$

$$xy=4y-2$$

$$xy-4y=-2$$

$$y(x-4)=-2$$

$$y=\frac{-2}{x-4}$$

$$f^{-1}(x)=\frac{-2}{x-4}$$

D

Radical Equation

A radical equation is an equation that has a variable in a radicand or has a variable expression with a rational exponent

$$y=\sqrt{x-h}+k \quad \text{or} \quad y=(x-h)^{\frac{a}{b}}+k$$

Domain	Range
$x\ge h$	$y\ge k$
$[h,\infty)$	$[k,\infty)$

Example

Find the domain of $f(x)=\sqrt{3x+6}$

Solution

$$\text{Let } 3x+6\ge 0$$

$$3x\ge -6$$

$$x\ge -2$$

Example

Find the domain of $f\left(x\right)=\dfrac{4x-3}{\sqrt{2x-8}}$

The restrictions of this function can be found by letting the radicand $2x-8\geq 0$ but the radical expression is in the denominator so the domain should be $2x-8>0$ and exclude zero from the domain $\longrightarrow 2x-8>0$

$$2x>8$$
$$x>4$$
$$\left[4,\infty\right)$$

56. Find the domain of the function

A $[0,\infty)$ **B** $[3,\infty)$

C $[1,\infty)$ **D** $[9,\infty)$

Let the radicand

$3X - 9 \geq 0$

$3X \geq 9$

$X \geq 3$

$$f\left(x\right)=\sqrt{3x-9}$$

B

57. Find the inverse of $f\left(x\right)=\sqrt{x+5}$

A x^2+5 **B** $\dfrac{5}{x^2}$

C x^2-5 **D** $\dfrac{-5}{x^2}$

$$f\left(x\right)=\sqrt{x+5}$$
$$y=\sqrt{x+5}$$
$$x=\sqrt{y+5}$$
$$\left(x\right)^2=\left(\sqrt{y+5}\right)^2$$
$$x^2=y+5$$
$$y=x^2-5$$
$$f^{-1}\left(x\right)=x^2-5$$

C

58. Find the domain of $f f\left(x\right)$ if $f\left(x\right)=\sqrt{x-16}$

A $R-\left\{\pm 16\right\}$ **B** $[16,\infty)$

C R **D** $\left[0,\infty\right)$

$$f\left(x\right)=\sqrt{x-h}+k$$
$$f\left(x\right)=\sqrt{x-16}+0 \longrightarrow k=0$$

Since range the of $f\left(x\right)$ is $y\geq 0$, $\left[0,\infty\right)$

Therefore the domain of $f^{-1}\left(x\right)$ is $x\geq 0$, $\left[0,\infty\right)$

D

Recall:

	Original Function	**Inverses Function**
	$f\left(x\right)$	$f^{-1}\left(x\right)$
Range	$x\geq 16$	$x\geq 0$
Domain	$y\geq 0$	$y\geq 16$

59. Rationalize the denominator $\dfrac{2}{\sqrt{18}+4}$

A $\sqrt{18}-4$ **B** $2\sqrt{18}-4$

C $\dfrac{2\sqrt{18}-4}{7}$ **D** $\dfrac{\sqrt{18}-4}{7}$

$$\frac{2}{\sqrt{18}+4}=\frac{2}{\sqrt{18}+4}\times\frac{\sqrt{18}-4}{\sqrt{18}-4}$$
$$=\frac{2\left(\sqrt{18}-4\right)}{18-16}$$
$$=\frac{2\left(\sqrt{18}-4\right)}{2}$$
$$=\sqrt{18}-4$$

A

Radical Rules

If the index of the radical expression is even and the exponent of the radicand is even too but the output of the radical expression is odd then we have to use absolute value.

Example

$$\sqrt[4]{\left(a+b\right)^{12}}=\left|\left(a+b\right)\right|^{\frac{12}{4}}$$
$$=\left|a\ b\right|^{3}$$

Note: If the operation between the radicand variables is addition or subtraction then we cannot distribute the root.

Example

$$\sqrt[n]{a+b}\neq\sqrt[n]{a}+\sqrt[n]{b}$$

$$\sqrt[n]{a-b}\neq\sqrt[n]{a}-\sqrt[n]{b}$$

60. Find the radical expression of $(x+y)^{\frac{3}{5}}$

A $\sqrt[3]{(x+y)^5}$ B $\sqrt[5]{(x+y)^3}$

C $\sqrt[3]{x^3}+\sqrt[5]{y^3}$ D $\sqrt[3]{x^5}+\sqrt[3]{y^5}$

Since $\sqrt[a]{x^b} = x^{\frac{a}{b}}$

and vice versa then $(x+y)^{\frac{3}{5}} = \sqrt[5]{(x+y)^3}$

B

61. Evaluate $\sqrt[8]{16}\cdot\sqrt{8}$

A 4 B 2

C 6 D 8

$$\begin{aligned}\sqrt[8]{16}\cdot\sqrt{8} &= 16^{\frac{1}{8}}\cdot 8^{\frac{1}{2}}\\ &= \left(2^4\right)^{\frac{1}{8}}\ \left(2^3\right)^{\frac{1}{2}}\\ &= 2^{\frac{4}{8}}\ 2^{\frac{3}{2}}\\ &= 2^{\frac{1}{2}}\ 2^{\frac{3}{2}}\\ &= 2^{\frac{1}{2}+\frac{3}{2}}\\ &= 2^2\\ &= 4\end{aligned}$$

A

62. Simplify $\sqrt[3]{x^{21}y^6}$

A $x^{\frac{1}{7}}y^{\frac{1}{2}}$ B x^7y^2

C $|x^7y^2|$ D $\sqrt{x^7y^2}$

$$\begin{aligned}\sqrt[3]{x^{21}y^6} &= \sqrt[3]{x^{21}}\cdot\sqrt[3]{y^6}\\ &= x^{\frac{21}{3}}\cdot y^{\frac{6}{3}}\\ &= x^7y^2\end{aligned}$$

B

63. Simplify $\sqrt[6]{64(x+5)^{18}}$

A $2(x+5)^3$ B $4|x+5|^3$

C $4(x+5)^3$ D $2|x+5|^3$

$$\begin{aligned}\sqrt[6]{64(x+5)^{18}} &= \sqrt[6]{64}\cdot\sqrt[6]{(x+5)^{18}}\\ &= 64^{\frac{1}{6}}\ |x+5|^{\frac{18}{6}}\\ &= 2|x+5|^3\end{aligned}$$

D

64. Simplify $\sqrt{49\,a^6\,b^{12}}$

A $7|a|^3\,b^6$ B $7a^3b^6$

C $7a^6b^{12}$ D $7|ab|^2$

$$\begin{aligned}\sqrt{49\,a^6\,b^{12}} &= \sqrt{49}\cdot\sqrt{a^6}\cdot\ \sqrt{b^{12}}\\ &= 7|a|^3\,b^6\end{aligned}$$

A

65. If $X \neq$ 0 then evaluate

$$\sqrt{\frac{\sqrt[3]{(x+3)^4\left(x^2+6x+9\right)}}{\sqrt[3]{\frac{x^{10}}{x^4}}}}$$

A $\dfrac{\sqrt{x+3}}{\sqrt{x}}$ B $\dfrac{x+3}{x}$

C $\dfrac{(x+3)^2}{x^2}$ D $\dfrac{x+3}{\sqrt{x}}$

$$\begin{aligned}\sqrt{\frac{\sqrt[3]{(x+3)^4\left(x^2+6x+9\right)}}{\sqrt[3]{\frac{x^{10}}{x^4}}}} &= \sqrt{\frac{\sqrt[3]{(x+3)^4}\cdot\sqrt[3]{(x+3)^2}}{\sqrt[3]{x^6}}}\\ &= \sqrt{\frac{(x+3)^{\frac{4}{3}}\cdot(x+3)^{\frac{2}{3}}}{x^{\frac{6}{3}}}}\\ &= \sqrt{\frac{(x+3)^{\frac{4+2}{3}}}{x^2}}\\ &= \sqrt{\frac{(x+3)^2}{x^2}}\\ &= \frac{\sqrt{(x+3)^2}}{\sqrt{x^2}}\\ &= \frac{x+3}{x}\end{aligned}$$

B

66. Solve $\sqrt{x+3}+2=8$

A 33 B 36

C 6 D 18

$$\begin{aligned}\sqrt{x+3}+2 &= 8\\ \sqrt{x+3} &= 6\\ \left(\sqrt{x+3}\right)^2 &= 6^2\\ x+3 &= 36\\ x &= 33\end{aligned}$$

A

Solving Radical Expressions

Step 1: Isolate the radical expression

Step 2: Square both sides

Step 3: Solve for X

67. Determine an interval in which one of the real zeros of $f(x)=\sqrt{x^2+7}=7$ might be

A $[4,5]$ B $[5,6]$ C $[6,7]$ D $[7,8]$

$$\begin{aligned}\sqrt{x^2+7} &= 7\\ \left(\sqrt{x^2+7}\right)^2 &= 7^2\\ x^2+7 &= 49\\ x^2 &= 42\end{aligned}$$

$$\begin{aligned}\sqrt{x^2} &= \sqrt{42}\\ x &= \sqrt{42}\\ \sqrt{36} < \sqrt{42} &< \sqrt{49}\\ 6 < \sqrt{42} &< 7\\ &[6,7]\end{aligned}$$

C

68. Solve the inequality $\sqrt{4x-4} > 6$

A $x > 10$ **B** $x < 10$

C $x > 4$ **D** $x < 4$

$$\sqrt{4x-4} > 6$$
$$\left(\sqrt{4x-4}\right)^2 > 6^2$$
$$4x-4 > 36$$
$$4x > 40$$
$$x > 10$$

➢ A

Adding Rational Expressions

To add or subtract two rational expressions with unlike denominators we find a new common denominator. But if they have like denominator then we simply add or subtract the numerators over the denominator.

Example

$$\frac{x^2+2x+3}{(x^2-9)} + \frac{4x+6}{(x^2-9)}$$
$$= \frac{x^2+2x+4x+6+3}{x^2-9}$$
$$= \frac{x^2+6x+9}{x^2-9}$$
$$= \frac{(x+3)^2}{(x-3)(x+3)}$$
$$= \frac{x+3}{x-3}$$

69. Simplify $\frac{7}{xy} - \frac{5}{y}$

A $\frac{7-5x}{xy}$ **B** $\frac{2}{x-y}$

C $\frac{2}{xy}$ **D** $\frac{7-5x}{x}$

$$\frac{7}{xy} - \frac{5}{y} = \frac{7}{xy} - \frac{5x}{xy}$$
$$= \frac{7-5x}{xy}$$

➢ A

70. Add $\frac{x+2}{x^2+3x+2} + \frac{x-3}{x^2-2x-3}$

A $\frac{1}{2(x+1)}$ **B** $\frac{2}{(x+1)}$

C $\frac{2}{2x+2}$ **D** $\frac{2x-1}{2x^2+x-1}$

➢ B

71. Simplify $\dfrac{1+\frac{1}{x}}{\;}$

A $\frac{x-1}{x+1}$ **B** $\frac{1}{x}$

C 1 **D** $\frac{x+1}{x-1}$

$$\left(1+\frac{1}{x}\right) \div \left(1-\frac{1}{x}\right) = \left(\frac{x}{x}+\frac{1}{x}\right) \div \left(\frac{x}{x}-\frac{1}{x}\right)$$
$$= \left(\frac{x+1}{x}\right) \div \left(\frac{x-1}{x}\right)$$
$$= \frac{x+1}{x} \times \frac{x}{x-1}$$
$$= \frac{x+1}{x-1}$$

➢ D

Vertical and Horizontal Asymptotes

For any $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, $a(x)$ and $b(x)$ have no common factor then $f(x)$ has:

1- Vertical asymptote (VA) if $b(x) = 0$

2- Horizontal asymptote (HA)

a- Degree of $a(x) <$ degree of $b(x)$ $\quad HA \to y = 0$

b- Degree of $a(x) =$ degree of $b(x)$

$$HA \to \frac{\text{coefficient of } a(x)}{\text{coefficient of } b(x)}$$

3- No horizontal asymptote if degree of $a(x) = b(x)$.

Special case of $f(x) = \frac{a(x)}{b(x)}$ if $a(x) = 1$ and $b(x) = x$

then it is called the parent function

$f(x) = \frac{1}{x}$ for $f(x) = \frac{1}{x-h} + k$

1- $f(x)$ is undefined if $x = h$

2- The vertical asymptote is at 3 $x = h$

- The horizontal asymptote is at $y = k$

72. Find the point where $f(x)$ **is undefined** $f(x) = \frac{1}{x+4} + 5$

A $x = 5$ **B** $x = 4$

C $x = -4$ **D** $x = -5$

$f(x)$ is in the form $f(x) = \frac{1}{x-h} + k$

therefore it is undefined at $x = h \to x = -4$

Or let the denominator $= 0 \to x + 4 = 0$

$x = -4$

➢ C

73. Find the vertical asymptote of $f(x)=\frac{13}{x+3}-5$

A $x=13$ **B** $x=-3$

C $x=3$ **D** $x=4$

Let the denominator $=0 \rightarrow x+3=0$

$x=-3$ **B**

74. Find the vertical asymptote of $f(x)=\frac{x-3}{x^2+7x+12}$

A $x=-4$ **B** $x=-4,\ x=-3$

C $x=3,\ x=4$ **D** $x=-3$

$$\frac{x-3}{x^2+7x+12}=\frac{x-3}{(x+3)(x+4)}$$

Since no common factor

Let the denominator =0

$(x+3)(x+4)=0$

$x+3=0$ or $x+4=0$

$x=-3$ $x=-4$

75. Find the vertical asymptote of $f(x)=\frac{x+3}{x^2+5x+6}$

A $x=2\ ,\ x=3$ **B** $x=-2\ ,\ x=-3$

C $x=-2$ **D** $x=-3$

$$f(x)=\frac{x+3}{x^2+5x+6}=\frac{x+3}{(x+2)(x+3)}$$

$$=\frac{1}{x+2}$$

let $x+2=0$

$x=-2$

Note: $x=-3$
is called a discontinuity

76. Find the horizontal asymptote of

$$f(x)=\frac{5x^3}{3x^3-2x^2-5}$$

A $y=\frac{5}{3}$ **B** $x=\frac{5}{3}$

C $x=\frac{3}{5}$ **D** $x=\frac{3}{5}$

Since the Degree of $a(x)=$ degree of $b(x)=$, then the horizontal asymptote is:

$$y=\frac{\text{coefficient of } a(x)}{\text{coefficient of } b(x)}$$

$$y=\frac{5}{3}$$ **A**

Variation

Type of variation	Explanation	Equation	Method of solving
Direct	As X increases y also increases As X decreases y also decreases	$y=kx$ $k=\frac{x}{y}$	$\frac{x_1}{y_1}=\frac{x_2}{y_2}$
Inverse	As X increases y also increases As X decreases y also decreases	$y=\frac{k}{x}$ $k=xy$	$x_1y_1=x_2y_2$
Joint	y varies directly to two or more quantities	$y=kxz$ $k=\frac{y}{xz}$	$\frac{y_1}{x_1z_1}=\frac{y_2}{x_2z_2}$
Combined	y varies directly to X and inversely to Z	$xy=kz$ $k=\frac{xy}{z}$	$\frac{x_1y_1}{z_1}=\frac{x_2y_2}{z_2}$

1.77 Describe the variation

y	3	4	7	10
x	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{14}$	$\frac{1}{20}$

A Direct **B** Inverse
C Joint **D** Combined

The equation of the inverse variation is $k = yx$ and if you multiply all values of y by its corresponding x values you will get a constant value $k = \frac{1}{2}$, then it's inverse variation.

➢ **B**

78. **If y varies inversely with x and $y = 4$ when $x = 3$, then find the value of x when $y = 6$**

A 2 **B** 6
C 1 **D** 4

$x_1 = 3$
$y_1 = 4$
$x_2 = ?$
$y_2 = 6$

x, y varies inversely

$x_1 y_1 = x_2 y$

$3 \times 4 = x \times 6$

$12 = 6x$

$x = 2$

➢ **A**

79. **The variable y varies jointly with x and z also $y = 70$ when $x = 4$ and $z = 10$ find the of y when $x = 8$ and $z = 2$**

A 10 **B** 48
C 50 **D** 28

$x_1 = 4$, $y_1 = 70$, $z_1 = 10$
$x_2 = 8$, $y_2 = ?$, $z_2 = 2$

Jointly variation $\rightarrow$ $\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$

$\frac{70}{4 \times 10} = \frac{y_2}{8 \times 2}$

$4 \times 10 \times y_2 = 8 \times 2 \times 70$

$y_2 = \frac{8 \times 2 \times 70}{4 \times 10}$

$= 28$

➢ **D**

80. **If y directly with x, and inversely with z. also $z = 20$ when $y = 4$ and $x = 2$, then find z when $x = 10$ and**

A 10 **B** 80
C −80 **D** −10

$x_1 = 2$ $x_2 = 10$
$y_1 = 4$ $y_2 = -5$
$z_1 = 20$ $z_2 = ?$

combined variation $\rightarrow$ y directly with $x \rightarrow \frac{y}{x}$
y inversly with $z \rightarrow yz$

$\rightarrow \frac{y_1 z_1}{x_1} = \frac{y_2 z_2}{x_2}$

$\rightarrow \frac{4 \times 20}{2} = \frac{-5 \times z_2}{10}$

$\rightarrow \frac{4 \times 20}{2} = \frac{-5 \times z_2}{10}$

$\rightarrow z_2 = \frac{10 \times 4 \times 20}{-5 \times 2}$

$\rightarrow z_2 = -80$ ➢ **C**

81. **If y varies directly with x, and $y = 8$ when $x = 24$, then find y when $x = 48$**

A 3 **B** 16
C 18 **D** 4

$x_1 = 24$
$y_1 = 8$
$x_2 = 48$
$y_2 = ?$

$\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\frac{24}{8} = \frac{48}{y_2}$

$y_2 = \frac{8 \times 48}{24}$

$= 8 \times 2$

$= 16$

➢ **B**

Extra Resource

LADDER METHOD

(Finding the LCM and GCF)

<u>Use the Ladder Method to find the LCM and GCF:</u>

<u>Step 1:</u> Write the numbers side by side with an L around it.

$$\begin{array}{|cc} 24 & 36 \\ \hline \end{array}$$

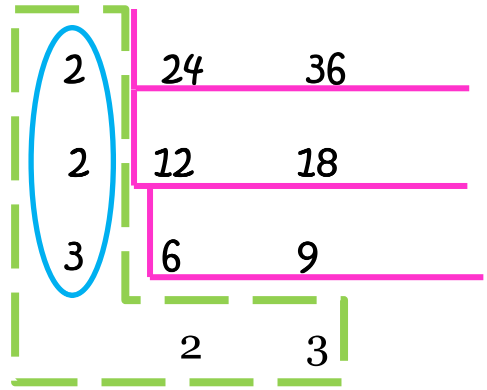
<u>Step 2:</u> Think of a common factor and write it on the left side of the L. Divide the numbers inside the L by the common factor and write the quotients <u>UNDER</u> the numbers.

$$\begin{array}{c|cc} 2 & 24 & 36 \\ \hline & 12 & 18 \end{array}$$

<u>Step 3</u>: If nothing goes into BOTH of the quotients evenly, go to step 4. If there is a common factor for the quotients, repeat step 2.

$$\begin{array}{c|cc} 2 & 24 & 36 \\ \hline 2 & 12 & 18 \\ \hline 3 & 6 & 9 \\ \hline & 2 & 3 \end{array}$$

Step 4: Use the numbers on the outside of the ladder to help you find the LCM and GCF :



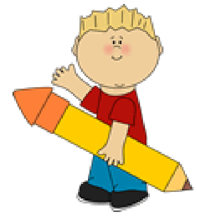
- **To find the LCM:** Multiply all of the numbers outside to the left and below the L, or "all around the L.

LCM = $2 \times 2 \times 3 \times 2 \times 3 = 72$

- **To find the GCF:** Multiply all of the numbers on the outside LEFT of the L.

GCF = $2 \times 2 \times 3 = 12$

<u>YOU TRY:</u> Find the LCM and GCF of the given numbers.



12, 18	36, 60
42, 60	32, 76